EVENTS AND OUTCOMES

The result of an experiment is called an

An **event** is

A **simple event** is an event that cannot be

The **sample space** is the set of

Example

If we flip a fair coin twice, describe the sample space, a simple event and compound event.

The sample space is the set of all possible simple events: { , , , }

Example of a Simple event:
We flip two tails: { }

Example of a Compound event:
The first flip is a head: { , }

BASIC PROBABILITY

Given that all outcomes are equally likely, we can compute the probability of an event E using this formula:

P(E)=

Example

If we select a card from a standard deck of 52 cards, calculate:

P(picking a 5) =

Example

If we randomly select a card from a standard deck of 52 playing cards, calculate:

P(♥) =

P(face) =

Question

If we randomly draw a marble from a bag containing 5 red marbles, 3 blue marbles, and 2 green marbles, calculate:

P(drawing a red marble)

P(drawing a green or blue marble)

Question

At some random moment, you glance at a calendar in the month of October.

a. What is the probability that the day is the 10th?

b. What is the probability that the day is the 10th or after?

Question

Compute the probability of randomly drawing one card from a deck and getting a Queen.

CERTAIN AND IMPOSSIBLE EVENTS

An ____ event has a probability of 0.

A certain event has a probability of ____.

The probability of any event must be:

____ ≤ ____ ≤ ____

CERTAIN AND IMPOSSIBLE EVENTS

The **complement of an event** is the event "____".

The notation E is used for the complement of event E.

We can compute the probability of the complement using ____

Notice also that

Question

What is the probability that a card drawn from a deck is not a Jack?

Question

A box contains 12 balls: 4 red, 5 blue, and 3 green. A ball is drawn randomly from the box. Find the probability of the following events:

The ball drawn is blue.
The probability is:

The ball drawn is not blue.
The probability is:

Question

What is the probability that Alice goes on vacation not in summer?
(Assume equal probability of each month and only one month is chosen)

INDEPENDENT EVENTS

Events A and B are **independent** events if the probability of Event B occurring is the same

Examples of independent events

Flipping a fair coin twice

Rolling a fair six-sided die and flipping a fair coin

Selecting a marble from a bag and then selecting another marble from the same bag with replacement

Question

Are the following events independent or dependent?

Randomly selecting two cards from a standard deck without replacement.

Question

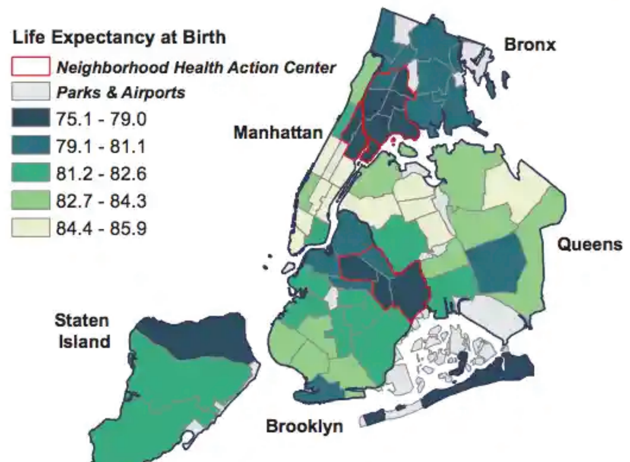
Are the following events independent or dependent?

Life expectancy and where you live in New York City.

The cohort life expectancy is the average life length of a particular cohort – a group of individuals born in a given year.

LIFE EXPECTANCY

Figure 4. Life Expectancy at Birth by Community District, New York City, 2006-2015



Source: Bureau of Vital Statistics, based on events occurring 2006-2015.
See Map of Community Districts and Boroughs, New York City on page 97.

- In 2015, New York City's life expectancy at birth was highest in Murray Hill (85.9), the Upper East Side (85.9), Battery Park/Tribeca (85.8), Greenwich Village/SOHO (85.8), and Elmhurst/Corona (85.6).
- In 2015, life expectancy at birth was lowest in Brownsville (75.1), Morrisania (76.2), Central Harlem (76.2), The Rockaways (76.5), and Bedford Stuyvesant (76.8).

P(A AND B) FOR INDEPENDENT EVENTS

If events A and B are independent, then the probability of both A and B occurring is:

$$P(A \text{ and } B)=P(A) \cdot P(B)$$

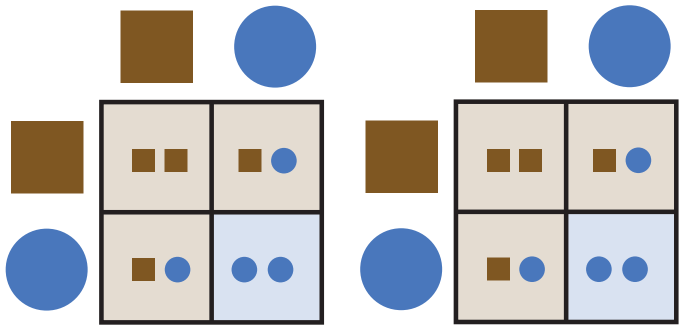
where P(A and B) is the probability of events A and B both occurring, P(A) is the probability of event A occurring, and P(B) is the probability of event B occurring.

Question

What is the probability of rolling a five followed by a six when rolling a die?

Question

What is the probabilty of the parents having one child with brown eyes and another child with blues eyes?



P(brown and blue)=P(brown) · P(blue)

$$= \frac{3}{4} \cdot \frac{1}{4}$$

$$= \frac{3}{16}$$

P(A OR B)

The probability of either A or B occurring (or both) is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example

In a group of 100 students, 60 students play tennis (event A) and 45 students play basketball (event B). Among them, 30 students play both tennis and basketball. What is the probability that a randomly selected student plays either tennis or basketball?

Example

In a group of 100 students, 60 students play tennis (event A) and 45 students play basketball (event B). Among them, 30 students play both tennis and basketball. What is the probability that a randomly selected student plays either tennis or basketball?

P(tennis or basketball)=

$$= \frac{60}{100} + \frac{45}{100} - \frac{30}{100} = \frac{75}{100}$$

Question

What is the probability that we draw either an odd numbered card in a deck of cards or a ten?

The probability the event B occurs, given that event A has happened, is represented as

This is read as:

example

Suppose we have a bag containing 3 red balls and 2 green balls.
We want to find the probability of drawing two red balls.
Let's denote the events as follows:

Event A: Drawing a red ball on the first draw.
Event B: Drawing a red ball on the second draw.

Given that we've drawn a red ball on the first draw, there are now 4 balls left in the bag, 2 of which are red and 2 are green.

Now, since we have 2 red balls and 4 balls total left in the bag, the probability of drawing a red ball on the second draw, given that the first ball drawn is red, is .

So the probability of B given A, denoted as P(B | A) is .

CONDITIONAL PROBABILITY FORMULA

If Events A and B are not independent, then

P(A and B) =

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

returning to previous example

Suppose we have a bag containing 3 red balls and 2 green balls.
We want to find the probability of drawing two red balls.
Let's denote the events as follows:

Event A: Drawing a red ball on the first draw.
Event B: Drawing a red ball on the second draw.

Given that we've drawn a red ball on the first draw, there are now 4 balls left in the bag, 2 of which are red and 2 are green.

Now, since we have 2 red balls and 4 balls total left in the bag, the probability of drawing a red ball on the second draw, given that the first ball drawn is red, is $\frac{2}{4}=\frac{1}{2}$.

So the probability of B given A, denoted as P(B | A) is $\frac{1}{2}$.

The probability of both events A and B occurring is:

$$P(A \cap B) = P(A) \cdot P(B \mid A) = \frac{2}{5} \cdot \frac{1}{2} = \frac{2}{10} = \frac{1}{5}$$

example

You have just developed a new COVID-19 diagnostic test with your team in the lab.

Event A represents the event that the person actually has COVID-19.
Event B represents the event that the COVID-19 test comes back positive.

The prevalence of COVID-19 in a certain population might be P(A)=0.02, meaning that 2% of people in the population have COVID-19.

The **sensitivity** of the COVID-19 test might be **P(B|A)=0.95**, meaning that given a person has COVID-19, there is a 95% chance that the COVID-19 test will come back positive.

Q:What is the probability of that the person has COVID-19 and that the test comes back as positive?

question

Suppose a health study examines the relationship between exercise frequency and the likelihood of developing certain health conditions. The data collected is summarized in the table below:

	No health condition
Exercises regularly	800
Does not exercise regularly	300
Total	1100

Q. What is the probability that a randomly chosen individual has a health condition given that they exercise regularly?

question

Suppose a health study examines the relationship between exercise frequency and the likelihood of developing certain health conditions. The data collected is summarized in the table below:

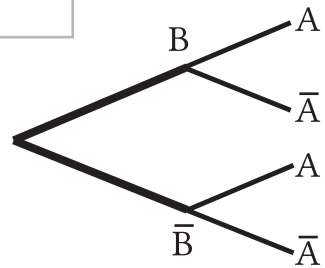
	No health condition
Exercises regularly	800
Does not exercise regularly	300
Total	1100

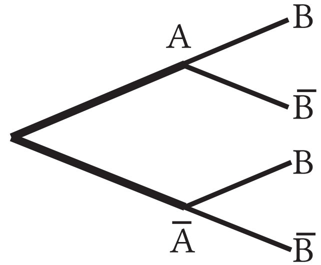
Q. What is the probability that a randomly chosen individual exercises regularly given that they have a health condition?

BAYES' THEOREM

If Events A and B are not independent, then

$$ = \frac{P(A)\,P(B \mid A)}{P(A)\,P(B \mid A) + P(\overline{A})\,P(B \mid \overline{A})}$$





example

In a certain community, the prevalence of individuals being in close contact with an infected individual of a certain disease is 20%.
Among those who have been in close contact with an infected individual, the probability of contracting the disease is 30%.
Among those who have **not** been in close contact with an infected individual, the probability of contracting the disease is 10%.
If a randomly selected individual from the community is found to have the disease, what is the probability that they had close contact with an infected individual?

$P(A) =$ $\qquad$ $P(\bar{A}) =$

$P(B \mid A) =$

$P(B \mid \bar{A}) =$

$$P(A \mid B) = \frac{P(A)\,P(B \mid A)}{P(A)\,P(B \mid A) + P(\bar{A})\,P(B \mid \bar{A})}$$

$$=$$

question (Monty Hall Problem)

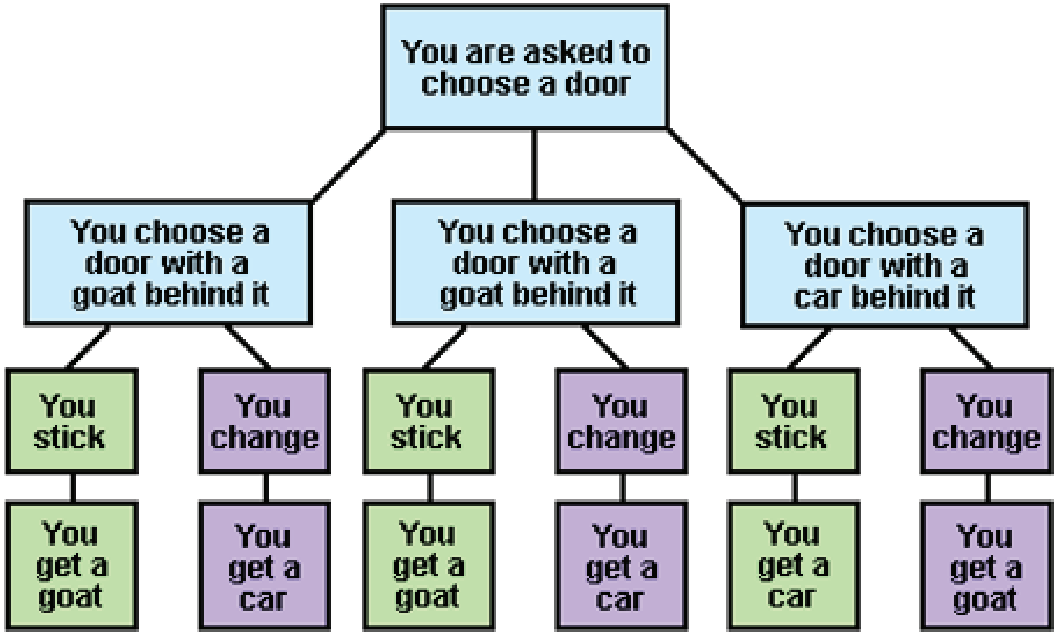
Suppose you are a contestant on a game show. The game involves three doors.
Behind one of the doors is a car, and behind the other two doors are goats.
You pick a door, say Door 1, but before it's opened, the host, who knows what's behind each door, opens another door, say Door 3, revealing a goat. Now, the host offers you the opportunity to switch your choice to Door 2. Should you switch?

A: The car is behind Door 1 (your initial choice).
B: The host opens Door 3 to reveal a goat.

The probability of the car being behind any specific door initially is $P(A) =$. Given that the car is behind Door 1, the probability that the host opens Door 3, revealing a goat is $P(B|A)=1$, because the host will always open a door with a goat behind it. Given that the car is **not** behind Door 1, the probability that the host opens Door 3 (revealing a goat) is $P(B|A)=1$, because the host will always open a door with a goat behind it.

$$P(A \mid B) = \frac{P(A)\,P(B \mid A)}{P(A)\,P(B \mid A) + P(\bar{A})\,P(B \mid \bar{A})} = \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1 + \frac{2}{3}\cdot 1} = \frac{1}{3}$$

So the probability that the car is behind Door 1 is unaffected by the host opening Door 3. But the car has to be behind either Door 1 or Door 2, so P(winning the car if you change) =


You are asked to choose a door
You choose a door with a goat behind it
You choose a door with a goat behind it
You choose a door with a car behind it
You stick
You change
You stick
You change
You stick
You change
You get a goat
You get a car
You get a goat
You get a car
You get a car
You get a goat

1. A card is drawn at random from a standard 52-card deck. For each of the following events, calculate the probability:
   (a) The card drawn is an Ace. The probability is: ________________
   (b) The card drawn is not a face card (not a Jack, Queen or King). The probability is: ________________
   (c) The card drawn is not a red card. The probability is: ________________

2. A card is drawn at random from a standard 52-card deck. Determine the probability for each of the following scenarios:
   (a) The card drawn is a Queen. The probability is: ________________
   (b) The card drawn is a black card. The probability is: ________________
   (c) The card drawn is not a heart card. The probability is: ________________

3. A box contains 10 green balls numbered 1 to 10 and 6 yellow balls numbered 1 to 6. A ball is drawn at random from the box. Find the probability of the given event.
   (a) The ball is green.
   (b) The ball is numbered greater than 5.
   (c) The ball is green or numbered greater than 5.
   (d) The ball is yellow and numbered less than 4.

4. A container holds 8 orange balls numbered 1 to 8 and 5 purple balls numbered 1 to 5. A ball is drawn at random from the container. Find the probability of the given event.
   (a) The ball is orange.
   (b) The ball is numbered less than 4.
   (c) The ball is orange or numbered less than 4.
   (d) The ball is purple and odd-numbered.

5. Suppose a jar contains 15 green marbles and 20 yellow marbles. If you reach in the jar and pull out 2 marbles at random at the same time, find the probability that both are green.

6. Suppose a box contains 12 black balls and 18 white balls. If you reach into the box and pull out 2 balls at random at the same time, find the probability that both are black.

7. A survey was conducted among a group of employees to assess their job satisfaction, summarized below:

	Satisfied	Neutral	Dissatisfied	Total
Male	25	10	5	40
Female	20	15	5	40
Total	45	25	10	80

If one employee is chosen at random, find the probability that the employee was male OR reported being satisfied.

8. In a recent cooking class, the preferences of students for different cuisines were recorded as follows:

	Italian	Mexican	Szechuan	Total
Male	12	8	10	30
Female	15	10	5	30
Total	27	18	15	60

If one student is chosen at random, find the probability that the student was female OR preferred Szechuan cuisine.

9. A local theater group held auditions for a play, and the results based on callbacks by role and gender are summarized below:

	Lead	Supporting	Extras	Total
Male	10	15	11	36
Female	12	10	6	28
Total	22	25	17	64

Table 1: Audition Callbacks by Gender

If one actor is chosen at random from those who auditioned, find the probability that the actor received a callback for Supporting roles GIVEN they are male.

10. A local fitness center conducted a survey to understand members' workout preferences. The results are summarized below:

	Cardio	Strength	Yoga	Total
Male	25	15	5	45
Female	20	10	15	45
Total	45	25	20	90

Table 2: Workout Preferences by Gender

If one member is chosen at random from those surveyed, find the probability that the member prefers Strength training GIVEN they are female.

11. A rare virus has an incidence rate of 0.05%. If the false negative rate is 6% and the false positive rate is 3%, compute the probability that a person who tests positive actually has the virus.

12. In a factory, a certain defect occurs in 0.3% of the products. If the false negative rate for detecting the defect is 7% and the false positive rate is 2%, compute the probability that a product which tests positive actually has the defect.

13. A particular medical condition has an incidence rate of 0.2%. If the false negative rate is 10% and the false positive rate is 4%, compute the probability that a person who tests positive actually has the condition.

14. In a city-wide recycling program, the incidence of correctly sorted recyclables is only 0.4%. If the false negative rate for the sorting machine is 9% and the false positive rate is 2%, compute the probability that an item flagged as recyclable actually is recyclable.

15. Consider a sensor designed to detect the presence of the critically endangered Amur leopard. The likelihood of encountering this species in a given area is 0.1%. The sensor has an 8% chance of missing an Amur leopard when one is present and a 5% chance of indicating the presence of an Amur leopard when it's not actually there. What is the probability that an area flagged by the sensor as having an Amur leopard truly contains one?

16. Consider a smoke detection system installed in a building. The actual incidence of a fire in this building is about 0.2%. The smoke detector has an 6% chance of failing to alert when there is a fire and a 9% chance of falsely signaling a fire when there isn't one. If the smoke detector indicates that there is a fire, what is the probability that a fire is actually present in the building?

17. Imagine a security alarm system installed in a museum to detect potential burglaries. The actual incidence of a burglary at the museum is about 0.1%. The alarm system has an 8% chance of failing to trigger when a burglary occurs and a 5% chance of sounding an alarm when there is no burglary. If the alarm goes off, what is the probability that a burglary is actually taking place in the museum?

18. Consider a cybersecurity system designed to detect data breaches in a healthcare facility. The actual rate of data breaches occurring is about 0.2%. The system has a 10% chance of failing to detect a breach when one happens and a 3% chance of falsely alerting when no breach has occurred. If the system indicates that a data breach has been detected, what is the probability that a breach is actually taking place?

19. Imagine a blood glucose monitoring device used by individuals with diabetes. The actual incidence of dangerously high blood glucose levels in this population is about 10%. The device has a 7% chance of failing to detect high glucose levels when they are present and a 5% chance of incorrectly indicating high levels when they are normal. If the device alerts that blood glucose levels are high, what is the probability that the levels are indeed elevated?

20. In his art class, a boy has 1 canvas, 1 type of paint, 5 brushes, and 3 different palettes. If he wants to create a new masterpiece, how many unique combinations can he use if he has to pick one of each item?

21. An avid gardener has a collection of plants. She has 1 type of pot, 1 kind of soil, 5 different seeds, and 3 types of fertilizer. How many unique plant arrangements can she create if she must use one of each item?

22. A party planner is organizing an event. She has 1 theme, 1 type of music playlist, 5 decorations, and 3 types of food. How many unique party setups can she create if she must choose one of each element?

23. At a dinner party, a host invites 7 guests. How many different ways can the guests be seated around the table if the arrangement matters?

24. An art curator has 9 paintings to display in an exhibition. How many different ways can the curator arrange the paintings on the wall?

25. A traveler has 6 cities to visit on a trip. How many different ways can they arrange the order of their visits to these cities?

26. In a talent show featuring 120 performers, the judges are set to award first, second, and third place as well as audience choice to the standout acts. How many different ways can the awards be given?

27. In a community garden, 150 members are entered into a raffle for a chance to win first, second, and third prizes. How many different ways can these prizes be assigned to the lucky winners?

28. A chef is planning a dinner menu and has 12 different recipes to choose from. She wants to select 5 recipes for the evening's meal. In how many ways can she choose the recipes?

29. A film festival director has 8 films submitted for screening. She needs to choose 3 films to feature in the festival. How many different ways can she select the films?

30. A gardener has 13 different types of flowers to choose from for her garden. She wants to plant 4 of them this season. In how many ways can she choose the flowers?

31. From a group of 10 volunteers, you randomly select 3 to help organize a community event. What is the probability that they are the 3 volunteers who signed up first?

32. In a team of 11 players, you randomly choose 2 for a special task. What is the probability that you select the goalkeeper and the captain?

33. In a country of 50 million residents, the number plates are of the form letter letter letter number number number, and all variations can be found in this country. What is the probability that the car that you drive in this country has a number plate with the first two letters XY?

34. In a raffle, participants select 6 tickets from a pool of 20. If a participant's tickets match all 6 drawn numbers, they win $50,000. If not, they lose $5. What is the expected value of participating in this raffle?

35. In a Wheel of Fortune game, a robot must fill in 6 letters to complete a phrase, choosing from the alphabet (A to Z). If the robot correctly identifies all 6 letters in the phrase, it wins $50,000. If it fails, it loses $1. What is the expected value of the robot participating in this game?

36. In a casino game, a player bets $10 on a spin of an American roulette wheel, which has 38 numbers (1-36, 0, and 00). If the ball lands on the player's chosen number, they win $350. If it lands on any other number, they lose their $10 bet. What is the expected value of playing this game?

37. A jar contains 3 red candies, 7 blue candies, and 15 green candies. Someone proposes a game: You randomly select one candy from the jar. If it is red, you win $5. If it is blue, you win $3. If it is green, you lose $2. What is your expected value if you play this game?