LINEAR GROWTH

If a quantity starts at size P_0 and grows by d every time period, then the quantity after n time periods can be determined using either of these relations:

____

$P_n = P_{n-1} + d$

____

$P_n = P_0 + d \cdot n$

In this equation, d represents the common difference – the amount that the population changes each time n increases by 1.

$$P_n = P_0 + d \cdot n$$

$$y = m \cdot x + c$$

Example

A coastal town recorded a dolphin population of 450 in 2012, and by 2018, the population had risen to 630. Assuming the dolphin population continues to increase at a steady rate, what is the expected population in 2025?

P0 = ________ P6 = ________

d = slope = ________ = ____ =

$P_n = P_0 + d \cdot n$, $P_n = 450 + 30d$ EXPLICIT

$P_0 = 450$, $P_n = P_{n-1} + 30$ RECURSIVE

Question

In a protected park, the number of oak trees was recorded at 3,200 in 2015. By 2020, the oak tree population had increased to 4,000. Assuming the tree population grows at a constant rate, what is the expected oak tree population in 2028?

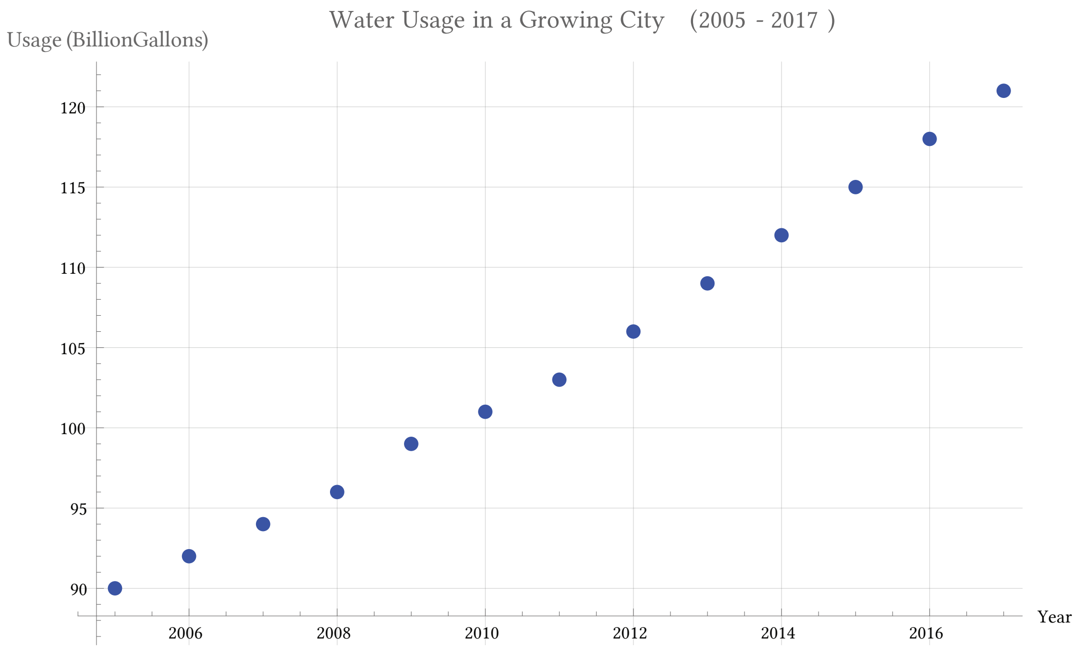
Question

A research station observed a penguin colony population of 1,500 in 2011, which grew to 1,950 by 2016. If the penguin population continues to grow at this same rate, what will the population be in 2023?

Example

The water consumption in a growing city has been increasing steadily. The data for water usage (in billions of gallons) from 2005 to 2017 is shown below. Find a model for this data and use it to
predict the water usage in 2025. If the trend continues, in what year will the water usage reach 220 billion gallons?

Year	’05	’06	’07	’08	’09	’10	’11	’12	’13	’14	’15	’16	’17
Usage (billion gallons)	90	92	94	96	99	101	103	106	109	112	115	118	121



Example

The cost, in dollars, of renting a storage unit for n months can be described by the explicit equation $C_n = 500 + 45n$. What does this equation tell us?

The value for C_0 in this equation is 500, so the initial starting cost is $____. This suggests there is an initial setup or administrative fee of $___ to start renting the unit. The value for d in the equation is 45, which means the cost increases by $__ each month. This tells us that the monthly rental fee for the storage unit is $45 per month.

Question

The number of women working in STEM fields in a certain country has been increasing over recent decades. Although the growth isn't perfectly linear, it is fairly consistent. Use the data from 1985 and 2015 to find an explicit formula for the number of women in STEM, then use it to estimate the number in 2025.

Year	1985	1990	2000	2010	2015
# of Women in STEM	15,000	18,250	26,500	34,700	39,800

Question

Consider a population of a species of birds that grows according to the recursive rule $P_n = P_{n-1} + 120$, with an initial population of $P_0 = 200$. Then:

$P_1 =$ P2 =

Find an explicit formula for the population. Your formula should involve n (use lowercasen).

$P_n =$

Use your explicit formula to find P_{50} .

$P_{50} =$

Question

A young tree is currently 5 feet tall, and it is expected to grow 1.5 feet each year. Create a linear growth model, with n=0 representing the current height of the tree.

EXPONENTIAL GROWTH

If a quantity starts at size P_0 and grows by R% (written as a decimal, r) every time period, then the quantity after n time periods can be determined using either of these relations:

Recursive form: $P_n = (1+r)\cdot P_{n-1}$

Explicit form: $P_n = (1+r)^n \cdot P_0$ or $P_n = P_0(1+r)^n$

We call r the _______________.

The term (1+r) is called the _________________, or _____________.

Example

Between 2015 and 2016, a small town in Oregon experienced a growth of approximately 4% to a population of 12,500 people. If this growth rate were to continue, what would the population of the town be in 2022?

First, we need to define the year corresponding to n=0.
Since we know the population in 2016, it makes sense to let 2016 correspond to n=___, so P_0 =________. The year 2022 would then be n=6. The growth rate is 4%, giving r=_____.

Using the explicit form:
$P_6 = (1 + \_\_\_\_\_)^{\_} \cdot 12{,}500 = (1.265319) \cdot 12{,}500 \approx$ _________

The model predicts that in 2022, the town would have a population of about _________ people.

Question

In a recent fiscal year, a local coffee shop reported revenues of $200,000, reflecting a growth of about 10% from the previous year. If this growth rate continues, what would the revenue of the coffee shop be in 2025?

Evaluating exponents on the calculator

To evaluate expressions like $(1.03)^6$, it will be easier to use a calculator than multiply 1.03 by itself six times. Most scientific calculators have a button for exponents. It is typically either labeled like:

^ , yx , or xy .

To evaluate 1.03^6 we'd type 1.03 ^ 6, or 1.03 y^x 6.

Try it out – you should get an answer around 1.1940523.

Question

Brazil is the fifth largest country in the world by area, with a population in 2024 of approximately 212 million people. The population is growing at a rate of about 0.4% each year. If this trend continues, what will Brazil's population be in 2050?

ROUNDING

A very small difference in the growth rates gets magnified greatly in exponential growth. For this reason, it is recommended to round the growth rate as little as possible.

If you need to round, keep at least three significant digits – numbers after any leading zeros. So 0.4162 could be reasonably rounded to 0.416. A growth rate of 0.001027 could be reasonably rounded to 0.00103.

Question

A community garden had 100 members in its first year and grew to 150 members by the end of its second year. If the garden is experiencing exponential growth and continues at the same rate, how many members should they expect five years after the garden's inception?

Question

A community garden had 100 members in its first year and grew to 150 members by the end of its second year. If the garden is experiencing **linear** growth and continues at the same rate, how many members should they expect five years after the garden's inception?

COMMON LOGARITHM

The **common logarithm**, written ________, undoes the exponential 10^x

This means that ___________, and likewise ____________

This also means the statement $10^a = b$ is equivalent to the statement $\log(b) = a$

log(x) is read as “log of x”, and means “the logarithm of the value x”. It is important to note that this is not multiplication – the log doesn’t mean anything by itself, just like √ doesn’t mean anything by itself; it has to be applied to a number.

Example

$\log(10) =$

$\log(100) =$

$\log(1{,}000) =$

$\log(10{,}000) =$

$\log(1) =$

$\log(1/10) =$

$\log(1/100) =$

$\log(1/1{,}000) =$

Question

Evaluate log(50) (use a calculator).

Solve $10^x = 100$

Solve $10^x = 56$

Solve $3(10^x) = 5$

Properties of Logs: Exponent Property

$$\log(A^r)=r\log(A)$$

Example

Rewrite log(36) using the exponent property for logs.

Question

Rewrite log(27) using the exponent property for logs.

Solving exponential equations with logarithms

1. ____________________. In other words, get it by itself on one side of the equation. This usually involves dividing by a number multiplying it.
2. Take the log of both sides of the equation.
3. Use the exponent property of logs to rewrite the exponential with the variable exponent multiplying the logarithm.
4. Divide as needed to solve for the variable.

Example

The town of Crestwood has been expanding its park area according to the equation $P_n = 320(1.04)^n$, where n represents the years after 2015, and the area P_n is measured in acres. In which year will Crestwood's park area reach 500 acres?

$\_\_\_\_ = \_\_\_(\_\_\_)^n$ $\quad \frac{500}{320} = (1.04)^n$

$1.5625 = (1.04)^n$

$\log(1.5625) = \log((1.04)^n)$

$\log(1.5625) = n \log(1.04)$

$n = \log(1.5625)/\log(1.04) \approx 11.38$

Question

A city's annual budget for public transportation has been growing according to the equation $P_n = 150(1.05)^n$, where n is the number of years after 2010, and the budget P_n is measured in millions of dollars. In what year will the budget reach 300 million dollars?

Question

An art restoration process involves applying protective coatings to an ancient painting to reduce fading. Each layer of coating blocks 80% of the remaining light exposure. If the painting originally received 200,000 lux of light exposure per month, how many layers are needed to reduce the light exposure to below 1,000 lux per month?

Question

A lake is treated to reduce invasive algae by adding a special chemical that removes 70% of the remaining algae with each application. If the lake starts with 5 million algae cells per liter, how many treatments are needed to bring the algae count down to 2,000 cells per liter?

CARRYING CAPACITY

The ________________________, or maximum sustainable population, is the largest population that an environment can support.

CARRYING CAPACITY

If a population is growing in a constrained environment with carrying capacity K, and absent constraint would grow exponentially with growth rate r, then the population behavior can be described by the logistic growth model:

$$P_n = P_{n-1} + r\left(1 - \frac{P_{n-1}}{K}\right)P_{n-1}$$

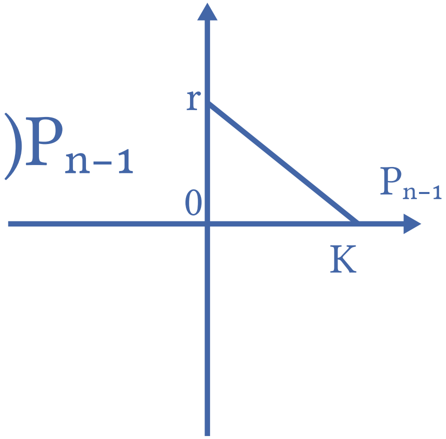
EXPONENTIAL GROWTH

$$P_n = (1 + r)\,P_{n-1}$$

$$P_n = P_{n-1} + r\,P_{n-1}$$

LOGISTIC GROWTH

$$P_n = P_{n-1} + r\left(1 - \frac{P_{n-1}}{K}\right)P_{n-1}$$



Example

An isolated island currently has a population of 300 wild goats. Ecologists estimate the island's ecosystem can support a maximum of 3,000 goats. Without any external factors, the goat population would increase by 40% per year. Predict the future population using the logistic growth model.

$$P_1 = P_{\_} + \_\_\_\_\_\left(1 - \frac{P_{\_}}{3000}\right)P_{\_} = 300 + 0.40\left(1 - \frac{300}{3000}\right)300 = 408$$

Using this to calculate the following year:

$$P_2 = P_{\_} + 0.40\left(1 - \frac{P_{\_}}{3000}\right)P_{\_} \approx 408 + 0.40\left(1 - \frac{408}{3000}\right)408 \approx 559$$

Question

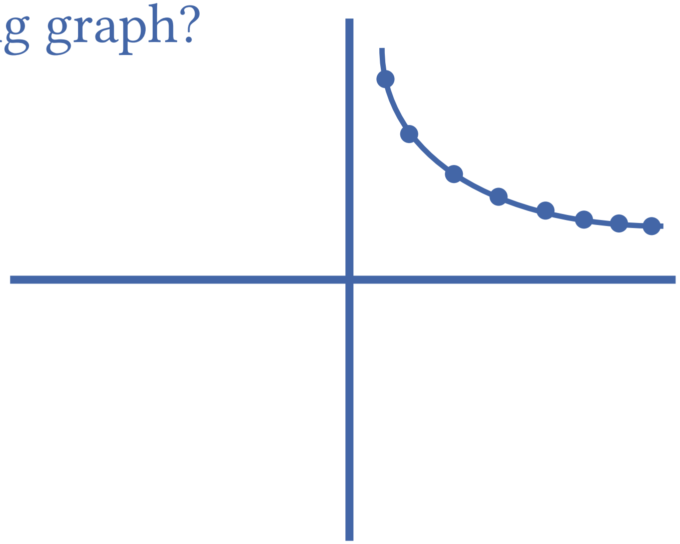
A fish tank currently houses 30 goldfish. Without any limitations, the number of goldfish would increase by 80% each year, but the tank can only support a maximum of 250 fish. Use the logistic growth model to predict the fish population in the next three years.

Question

A wildlife reserve currently has 45 deer. If left unregulated, the population would grow by 65% each year, but the reserve can only support a maximum population of 500 deer. Use the logistic growth model to predict the deer population in the next three years.

Question

What growth model best represents the following graph?



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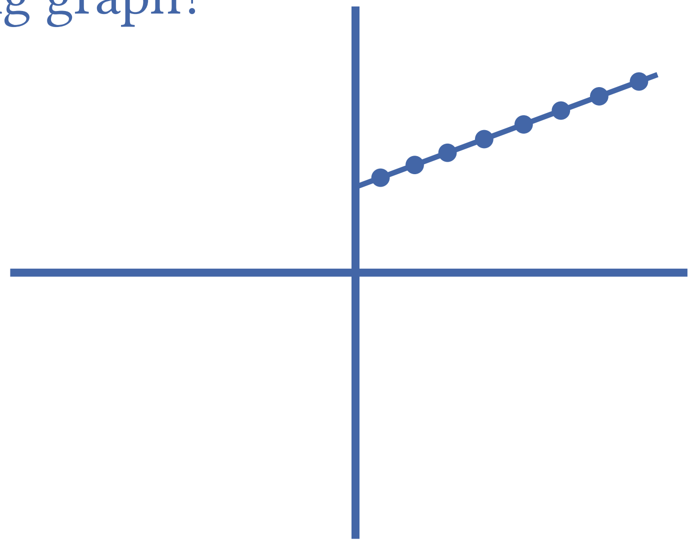

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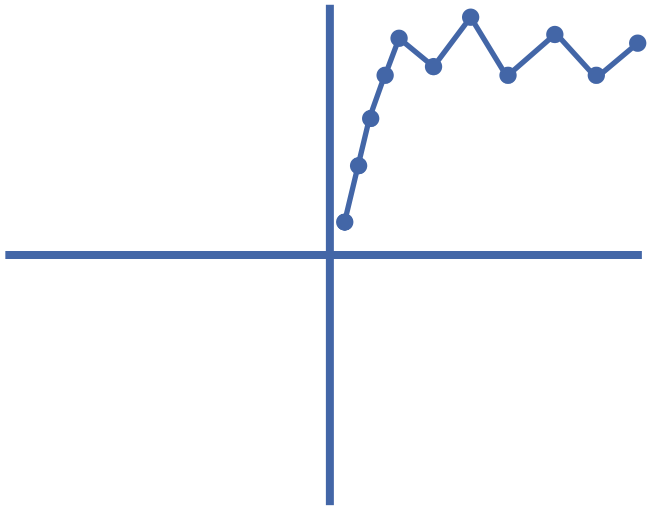

1. A car manufacturing plant introduces a new SUV model, and its sales follow a linear growth model. In the first quarter, the factory sells 1000 SUVs ($P_0 = 1000$). In the second quarter, the sales increase to 1200 SUVs ($P_1 = 1200$).

   Write the recursive formula for the number of SUVs sold, P_n, in the $(n+1)$th quarter.

   $$P_n = \_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_$$

   Write the explicit formula for the number of SUVs sold, P_n, in the $(n+1)$th quarter.

   $$P_n = \_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_ n$$

   If this trend continues, how many SUVs will be sold in the sixth quarter?

   __________ SUVs

2. A reservoir's water level is increasing according to a linear growth model. The initial water level (week 0) is $P_0 =$ ________ cubic meters, and the water level after 8 weeks is $P_8 =$ ________ cubic meters.

   Write an explicit formula for the water level in the reservoir after n weeks.

   $$P_n = \_\_\_\_\_\_\_\_ n + \_\_\_\_\_\_\_\_$$

   After how many weeks will the water level in the reservoir reach 2500 cubic meters? __________ weeks

3. A company has 50 computers in its office. To improve productivity, the company decides to upgrade its computer systems. As part of the upgrade plan, the company will add 4 new computers at the end of each month for the next 24 months.

   How many computers will the company have at the end of 18 months? __________

4. A colony of bacteria grows according to an exponential growth model. The initial population is $P_0 = 100$, and the growth rate is $r = 0.2$.

   Find an explicit formula for P_n. Your formula should involve n.

   Use your formula to find P_{11}.

   Give all answers accurate to at least one decimal place.

5. A forest ecosystem experiences growth according to an exponential model. At the beginning of the observation, the forest contains 500 trees ($P_0 = 500$) and after one year, it contains 700 trees ($P_1 = 700$).

   Complete the recursive formula: $P_n =$ ______ $\times P_{n-1}$

   Write an explicit formula for P_n: $P_n =$ __________

6. Find the logarithm without using your calculator.

   $\log_{10}(10) =$ ________________

7. Compute the following logarithm without using a calculator:

   $\log_2(8) =$ ________________

8. Calculate the logarithm without a calculator:

   $\log_3(27) =$ ________________

9. Compute the logarithm without using your calculator:

   $\log_5(0.2) =$ ________________

10. Express the equation in exponential form:

    $\log_2 16 = 4$.

    That is, write your answer in the form $2^A = B$. Then $A =$ ________ and $B =$ ________.

11. Express the equation in exponential form:

    $\log_3 81 = 4$.

    That is, write your answer in the form $3^A = B$. Then $A =$ ________ and $B =$ ________.

12. Express the equation in exponential form:

    $\log_4 64 = 3$.

    That is, write your answer in the form $4^A = B$. Then $A =$ ________ and $B =$ ________.

13. Solve correct to 2 decimal places:

    $2 \cdot 3^x = 18$

    $x =$ ___________

14. Solve correct to 2 decimal places:

    $3 \cdot 2^x = 12$

    $x =$ ___________

15. Solve correct to 2 decimal places:

    $4 \cdot 5^x = 80$

    $x =$ ___________

16. Solve correct to 2 decimal places:

    $5 \cdot 4^x = 100$

    $x =$ ___________

17. Consider a population of deer in a national park, whose growth is described by the logistic equation. It's estimated that the carrying capacity of the park is 2000 deer. In the absence of constraints, the population would grow by 150

    If the initial population is $p_0 = 800$ deer, then after one year the population of the park is:

    $p_1 =$ ___________

    After two years, the population of the park becomes:

    $p_2 =$ ___________

18. Consider a colony of bacteria in a laboratory culture, whose growth is described by the logistic equation. It's estimated that the carrying capacity of the culture is 5000 bacteria. In the absence of constraints, the population would grow by 200

    If the initial population is $p_0 = 100$ bacteria, then after one day the population of the culture is:

    $p_1 =$ ___________

    After two days, the population of the culture becomes:

    $p_2 =$ ___________

19. Consider a population of squirrels in a city park, whose growth is described by the logistic equation. It's estimated that the carrying capacity of the park is 1000 squirrels. In the absence of constraints, the population would grow by 150

    If the initial population is $p_0 = 200$ squirrels, then after one year the population of the park is:

    $p_1 =$ ___________

    After two years, the population of the park becomes:

    $p_2 =$ ___________

20. Let $P(t) = 3000(1.08)^t$ be the population of a town t years after the year 2000.

    Estimate in which year the population will reach 4300.

    Year = ____________

21. Let $P(t) = 2000(1.04)^t$ represent the population of a species of birds t years after the year 2020.

    Estimate in which year the population will reach 4000.

    Year = ____________