The probability the event B occurs, given that event A has happened, is represented as

This is read as:

example

Suppose we have a bag containing 3 red balls and 2 green balls.
We want to find the probability of drawing two red balls.
Let's denote the events as follows:

Event A: Drawing a red ball on the first draw.
Event B: Drawing a red ball on the second draw.

Given that we've drawn a red ball on the first draw, there are now 4 balls left in the bag, 2 of which are red and 2 are green.

Now, since we have 2 red balls and 4 balls total left in the bag, the probability of drawing a red ball on the second draw, given that the first ball drawn is red, is .

So the probability of B given A, denoted as P(B | A) is .

CONDITIONAL PROBABILITY FORMULA

If Events A and B are not independent, then

$$P(A \text{ and } B) =$$

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

returning to previous example

Suppose we have a bag containing 3 red balls and 2 green balls.
We want to find the probability of drawing two red balls.
Let's denote the events as follows:

Event A: Drawing a red ball on the first draw.
Event B: Drawing a red ball on the second draw.

Given that we've drawn a red ball on the first draw, there are now 4 balls left in the bag, 2 of which are red and 2 are green.

Now, since we have 2 red balls and 4 balls total left in the bag, the probability of drawing a red ball on the second draw, given that the first ball drawn is red, is $\frac{2}{4} = \frac{1}{2}$.

So the probability of B given A, denoted as P(B | A) is $\frac{1}{2}$.

The probability of both events A and B occurring is:

$$P(A \cap B) = P(A) \cdot P(B \mid A) = \frac{2}{5} \cdot \frac{1}{2} = \frac{2}{10} = \frac{1}{5}$$

example

You have just developed a new COVID-19 diagnostic test with your team in the lab.

Event A represents the event that the person actually has COVID-19.
Event B represents the event that the COVID-19 test comes back positive.

The prevalence of COVID-19 in a certain population might be P(A)=0.02, meaning that 2% of people in the population have COVID-19.

The **sensitivity** of the COVID-19 test might be **P(B|A)=0.95**, meaning that given a person has COVID-19, there is a 95% chance that the COVID-19 test will come back positive.

Q:What is the probability of that the person has COVID-19 and that the test comes back as positive?

question

Suppose a health study examines the relationship between exercise frequency and the likelihood of developing certain health conditions. The data collected is summarized in the table below:

	No health condition
Exercises regularly	800
Does not exercise regularly	300
Total	1100

Q. What is the probability that a randomly chosen individual has a health condition given that they exercise regularly?

question

Suppose a health study examines the relationship between exercise frequency and the likelihood of developing certain health conditions. The data collected is summarized in the table below:

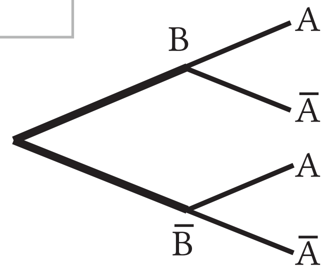
	No health condition
Exercises regularly	800
Does not exercise regularly	300
Total	1100

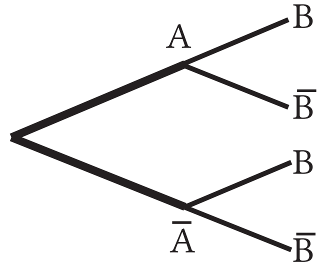
Q. What is the probability that a randomly chosen individual exercises regularly given that they have a health condition?

BAYES' THEOREM

If Events A and B are not independent, then

$$ = \frac{P(A)\,P(B \mid A)}{P(A)\,P(B \mid A) + P(\bar{A})\,P(B \mid \bar{A})}$$





example

In a certain community, the prevalence of individuals being in close contact with an infected individual of a certain disease is 20%.
Among those who have been in close contact with an infected individual, the probability of contracting the disease is 30%.
Among those who have **not** been in close contact with an infected individual, the probability of contracting the disease is 10%.
If a randomly selected individual from the community is found to have the disease, what is the probability that they had close contact with an infected individual?

$P(A) =$ $\qquad$ $P(\bar{A}) =$

$P(B \mid A) =$

$P(B \mid \bar{A}) =$

$$P(A \mid B) = \frac{P(A)\,P(B \mid A)}{P(A)\,P(B \mid A) + P(\bar{A})\,P(B \mid \bar{A})}$$

$=$

question (Monty Hall Problem)

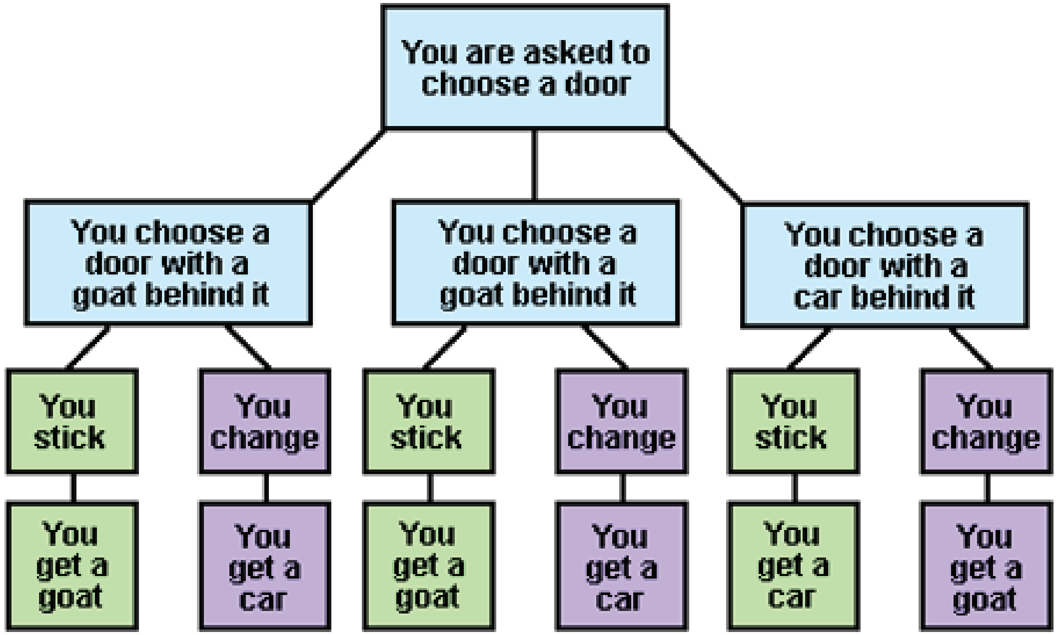
Suppose you are a contestant on a game show. The game involves three doors.
Behind one of the doors is a car, and behind the other two doors are goats.
You pick a door, say Door 1, but before it's opened, the host, who knows what's behind each door, opens another door, say Door 3, revealing a goat. Now, the host offers you the opportunity to switch your choice to Door 2. Should you switch?

A: The car is behind Door 1 (your initial choice).
B: The host opens Door 3 to reveal a goat.

The probability of the car being behind any specific door initially is P(A) = . Given that the car is behind Door 1, the probability that the host opens Door 3, revealing a goat is P(B|A)=1, because the host will always open a door with a goat behind it. Given that the car is **not** behind Door 1, the probability that the host opens Door 3 (revealing a goat) is P(B|A)=1, because the host will always open a door with a goat behind it.

$$P(A \mid B) = \frac{P(A)\,P(B \mid A)}{P(A)\,P(B \mid A) + P(\bar{A})\,P(B \mid \bar{A})} = \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1 \;+\; \frac{2}{3}\cdot 1} = \frac{1}{3}$$

So the probability that the car is behind Door 1 is unaffected by the host opening Door 3. But the car has to be behind either Door 1 or Door 2, so P(winning the car if you change) =


You are asked to choose a door
You choose a door with a goat behind it
You choose a door with a goat behind it
You choose a door with a car behind it
You stick
You change
You stick
You change
You stick
You change
You get a goat
You get a car
You get a goat
You get a car
You get a car
You get a goat