EVENTS AND OUTCOMES

The result of an experiment is called an **outcome**.

An **event** is any particular outcome or group of outcomes.

A **simple event** is an event that cannot be broken down further

The **sample space** is the set of all possible simple events.

Example

If we flip a fair coin twice, describe the sample space, a simple event and compound event.

The sample space is the set of all possible simple events: {HH, HT, TH, TT}

Example of a Simple event:
We flip two tails: {TT}

Example of a Compound event:
The first flip is a head: {HT, HH}

BASIC PROBABILITY

Given that all outcomes are equally likely, we can compute the probability of an event E using this formula:

$$P(E) = \frac{\text{Number of outcomes corresponding to the event E}}{\text{Total number of equally likely outcomes}}$$

Example

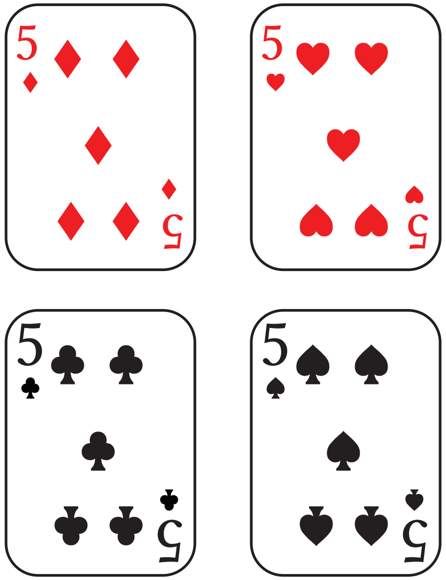
If we select a card from a standard deck of 52 cards, calculate:

P(picking a 5) =

Example

If we select a card from a standard deck of 52 cards, calculate:

$$P(\text{picking a 5}) = \frac{\text{number of 5's in the deck}}{\text{number of cards in the deck}}$$



$$= \frac{4}{52} = \frac{1}{13}$$

Example

If we randomly select a card from a standard deck of 52 playing cards, calculate:

P(♥) =

P(face) =

Example

If we randomly select a card from a standard deck of 52 playing cards, calculate:

$$P(\heartsuit) = \frac{\text{number of } \heartsuit \text{ in deck}}{\text{total number of cards in deck}}$$

$$= \quad 13/52 \quad = \quad 1/4$$

$$P(\text{face}) = \frac{\text{number of faces in deck}}{\text{total number of cards in deck}}$$

$$= \quad 12/52 = 3/13$$

Question

If we randomly draw a marble from a bag containing 5 red marbles, 3 blue marbles, and 2 green marbles, calculate:

P(drawing a red marble)

P(drawing a green or blue marble)

Question

If we randomly draw a marble from a bag containing 5 red marbles, 3 blue marbles, and 2 green marbles, calculate:

P(drawing a red marble) = number of red marbles

$$= \frac{\text{total number of marbles}}{}$$

P(drawing a green or blue marble) =

$$= \frac{\text{number of green and blue marbles} \quad \text{total number of marbles}}{}$$

Question

At some random moment, you glance at a calendar in the month of October.

a. What is the probability that the day is the 10th?

b. What is the probability that the day is the 10th or after?

Question

Compute the probability of randomly drawing one card from a deck and getting a Queen.

CERTAIN AND IMPOSSIBLE EVENTS

An **impossible** event has a probability of 0.

A **certain event** has a probability of 1.

The probability of any event must be:

$$0 \leq P(E) \leq 1$$

CERTAIN AND IMPOSSIBLE EVENTS

The **complement of an event** is the event
"E doesn't happen".

The notation E is used for the complement of event E.

We can compute the probability of the complement using $P(\bar{E}) = 1 - P(E)$
Notice also that $P(E)=1 - P(\bar{E})$

Question

What is the probability that a card drawn from a deck is not a Jack?

Question

A box contains 12 balls: 4 red, 5 blue, and 3 green. A ball is drawn randomly from the box. Find the probability of the following events:

The ball drawn is blue.
The probability is:

The ball drawn is not blue.
The probability is:

Question

What is the probability that Alice goes on vacation not in summer?
(Assume equal probability of each month and only one month is chosen)

INDEPENDENT EVENTS

Events A and B are **independent** events if the probability of Event B occurring is the same whether or not Event A occurs.

Examples of independent events

Flipping a fair coin twice

Rolling a fair six-sided die and flipping a fair coin

Selecting a marble from a bag and then selecting another marble from the same bag with replacement

Question

Are the following events independent or dependent?

Randomly selecting two cards from a standard deck without replacement.

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The probability of the second draw is dependent on the outcome of the first draw is a card has been removed and cannot be chosen again.

Question

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Life expectancy and where you live in New York City.

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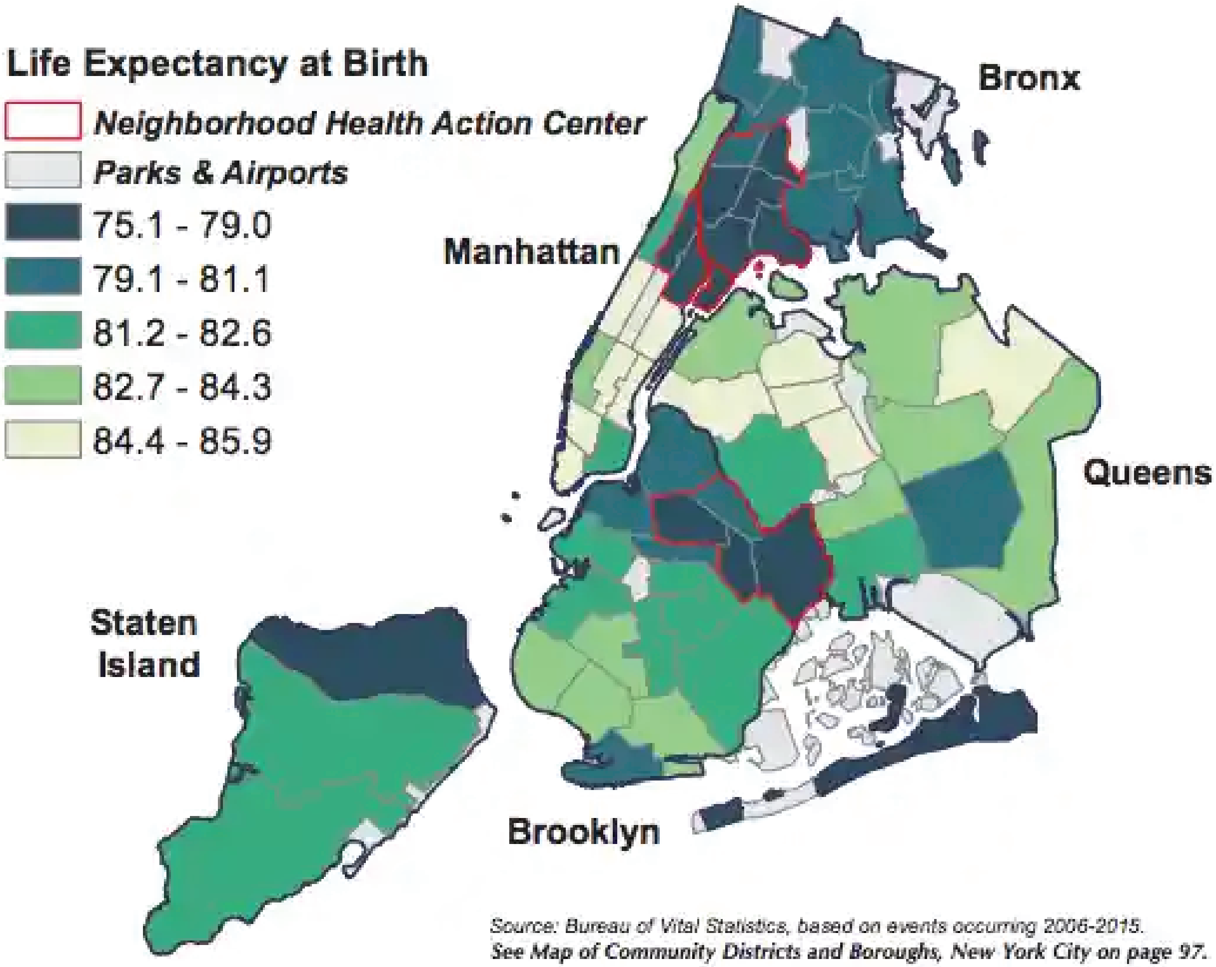
Life expectancy and where you live in New York City.

The cohort life expectancy is the average life length of a particular cohort – a group of individuals born in a given year.

LIFE EXPECTANCY

Figure 4. Life Expectancy at Birth by Community District, New York City, 2006-2015



Source: Bureau of Vital Statistics, based on events occurring 2006-2015.
See Map of Community Districts and Boroughs, New York City on page 97.

- In 2015, New York City's life expectancy at birth was highest in Murray Hill (85.9), the Upper East Side (85.9), Battery Park/ Tribeca (85.8), Greenwich Village/SOHO (85.8), and Elmhurst/Corona (85.6).
- In 2015, life expectancy at birth was lowest in Brownsville (75.1), Morrisania (76.2), Central Harlem (76.2), The Rockaways (76.5), and Bedford Stuyvesant (76.8).

P(A AND B) FOR INDEPENDENT EVENTS

If events A and B are independent, then the probability of both A and B occurring is:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

where P(A and B) is the probability of events A and B both occurring, P(A) is the probability of event A occurring, and P(B) is the probability of event B occurring.

Question

What is the probability of rolling a five followed by a six when rolling a die?

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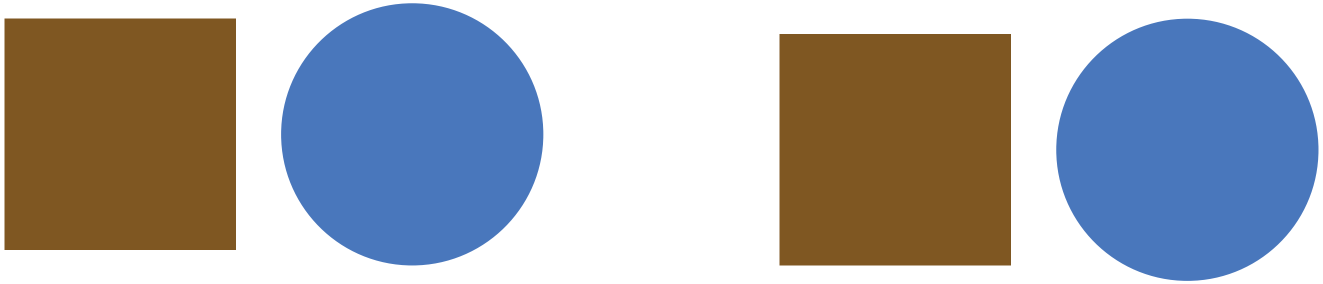
What is the probability of rolling a five followed by a six when rolling a die?

$$\begin{aligned} P(\text{five and six}) &= P(\text{five}) \cdot P(\text{six}) \\ &= \frac{1}{6} \cdot \frac{1}{6} \end{aligned}$$

$$\frac{1}{36}$$

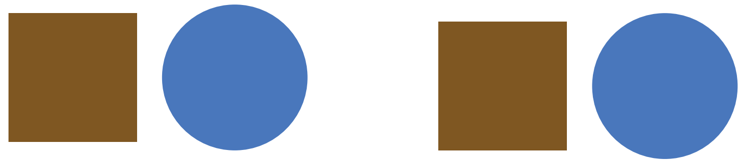
Question

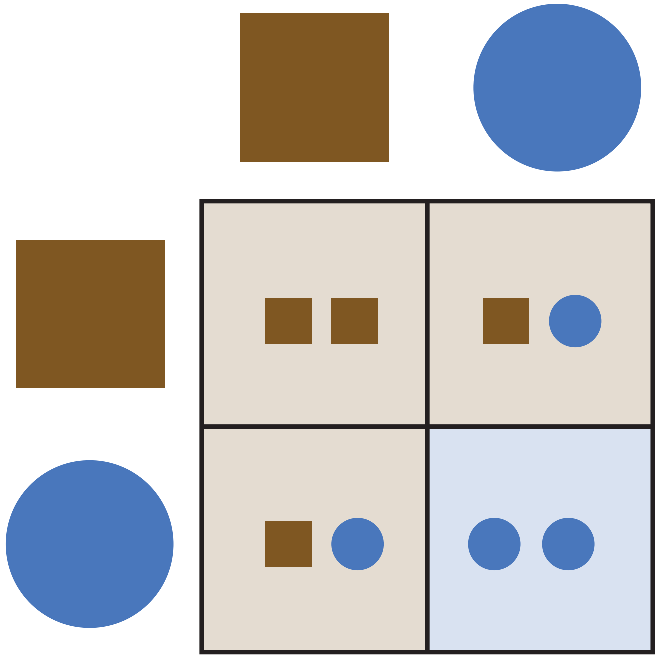
What is the probabilty of the parents

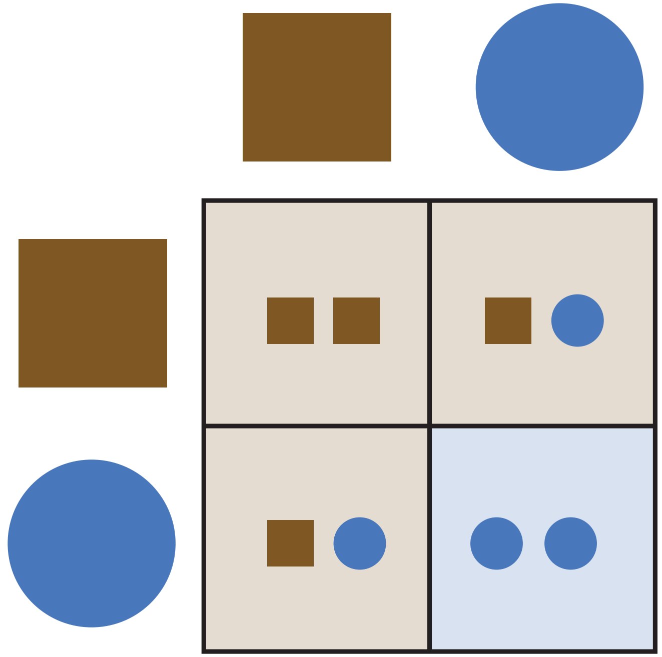


having one child with brown eyes and another child with
blues eyes?

Question

What is the probabilty of the parents  having one child with brown eyes and another child with blues eyes?





$$
\begin{aligned}
P(\text{brown and blue}) &= P(\text{brown}) \cdot P(\text{blue}) \\
&= \frac{3}{4} \cdot \frac{1}{4} \\
&= \frac{3}{16}
\end{aligned}
$$

P(A OR B)

The probability of either A or B occurring (or both) is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example

In a group of 100 students, 60 students play tennis (event A) and 45 students play basketball (event B). Among them, 30 students play both tennis and basketball. What is the probability that a randomly selected student plays either tennis or basketball?

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$$\text{P(tennis or basketball)} = \text{P(tennis)} + \text{P(basketball)} - \text{P(tennis and basketball)}$$

$$= \frac{60}{100} + \frac{45}{100} - \frac{30}{100} = \frac{75}{100}$$

Question

What is the probability that we draw either an odd numbered card in a deck of cards or a ten?

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What is the probability that we draw either an odd numbered card in a deck of cards or a ten?

$$P(\text{odd or } 10) = P(\text{odd}) + P(\text{ten}) - P(\text{odd and ten})$$

$$= \frac{20}{52} + \frac{4}{52} - \frac{0}{52} = \frac{24}{52} = \frac{6}{13}$$

MUTUALLY EXCLUSIVE

Two events A and B are mutually exclusive if

$$P(A \text{ or } B) = P(A) + P(B)$$

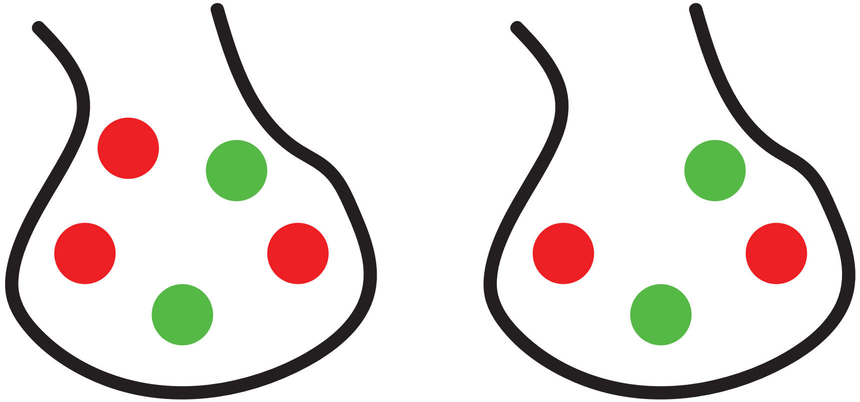
CONDITIONAL PROBABILITY

The probability the event B occurs, given that event A has happened, is represented as

$$P(B \mid A)$$

This is read as “the probability of B given A”

example



Suppose we have a bag containing 3 red balls and 2 green balls.
We want to find the probability of drawing two red balls.
Let's denote the events as follows:

Event A: Drawing a red ball on the first draw.
Event B: Drawing a red ball on the second draw.

Given that we've drawn a red ball on the first draw, there are now 4 balls left in the bag, 2 of which are red and 2 are green.

Now, since we have 2 red balls and 4 balls total left in the bag, the probability of drawing a red ball on the second draw, given that the first ball drawn is red, is $\frac{2}{4} = \frac{1}{2}$.

So the probability of B given A, denoted as $P(B \mid A)$ is $\frac{1}{2}$.

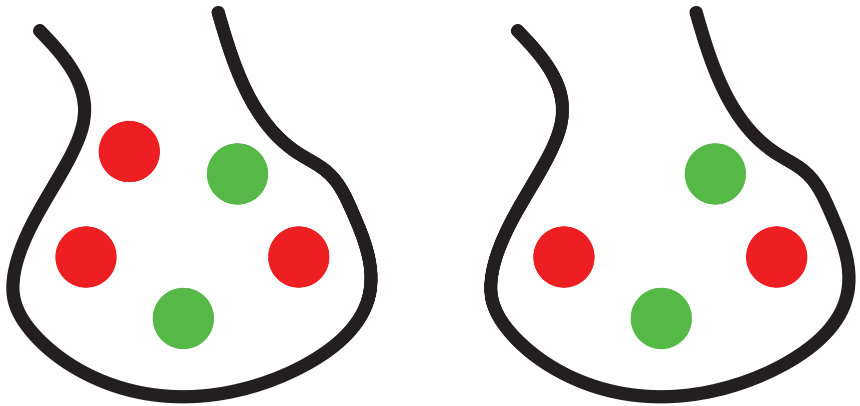
CONDITIONAL PROBABILITY FORMULA

If Events A and B are not independent, then

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

returning to previous example



Suppose we have a bag containing 3 red balls and 2 green balls.
We want to find the probability of drawing two red balls.
Let's denote the events as follows:

Event A: Drawing a red ball on the first draw.
Event B: Drawing a red ball on the second draw.

Given that we've drawn a red ball on the first draw, there are now 4 balls left in the bag, 2 of which are red and 2 are green. $P(A) = \frac{3}{5}$

Now, since we have 2 red balls and 4 balls total left in the bag, the probability of drawing a red ball on the second draw, given that the first ball drawn is red, is $\frac{2}{4} = \frac{1}{2}$.

So the probability of B given A, denoted as $P(B \mid A)$ is $\frac{1}{2}$.

The probability of both events A and B occurring is:

$$P(A \cap B) = P(A) \cdot P(B \mid A) = \frac{3}{5} \cdot \frac{1}{2} = \frac{3}{10}$$

example

You have just developed a new COVID-19 diagnostic test with your team in the lab.

Event A represents the event that the person actually has COVID-19.
Event B represents the event that the COVID-19 test comes back positive.

The prevalence of COVID-19 in a certain population might be $P(A)=0.02$, meaning that 2% of people in the population have COVID-19.

The sensitivity of the COVID-19 test might be $P(B|A)=0.95$, meaning that given a person has COVID-19, there is a 95% chance that the COVID-19 test will come back positive.

Q:What is the probability of that the person has COVID-19 and that the test comes back as positive?

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$$P(A \cap B) = P(A) \cdot P(B \mid A) = (0.02) \cdot (0.95) = 0.019$$

question

Suppose a health study examines the relationship between exercise frequency and the likelihood of developing certain health conditions. The data collected is summarized in the table below:

	No health condition	Has health condition	Total
Exercises regularly	800	200	1000
Does not exercise regularly	300	300	600
Total	1100	500	1600

Q. What is the probability that a randomly chosen individual has a health condition given that they exercise regularly?

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$$P(\text{has health condition} \mid \text{exercises regularly}) = \frac{200}{1000} = \frac{1}{5}$$

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BAYES' THEOREM

If Events A and B are not independent, then

$$P(A \mid B) = \frac{P(A)\,P(B \mid A)}{P(A)\,P(B \mid A) + P(\bar{A})\,P(B \mid \bar{A})}$$

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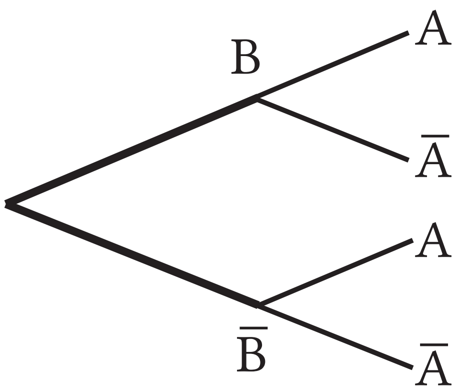

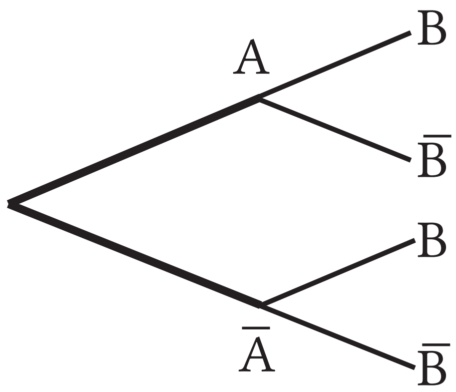

example

In a certain community, the prevalence of individuals being in close contact with an infected individual of a certain disease is 20%.
Among those who have been in close contact with an infected individual, the probability of contracting the disease is 30%. Among those who have **not** been in close contact with an infected individual, the probability of contracting the disease is 10%.
If a randomly selected individual from the community is found to have the disease,
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Event A: Individual had close contact with an infected individual.
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Event A: Individual had close contact with an infected individual.
Event B: Individual has the infectious disease.

$P(A) = 0.20$ $\quad$ $P(\overline{A}) = 0.80$
$P(B \mid A) = 0.30$
$P(B \mid \overline{A}) = 0.10$

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$$P(A \mid B) = \frac{P(A)\,P(B \mid A)}{P(A)\,P(B \mid A) + P(\overline{A})\,P(B \mid \overline{A})}$$

$P(A) = 0.20$ $\quad$ $P(\overline{A}) = 0.80$
$P(B \mid A) = 0.30$
$P(B \mid \overline{A}) = 0.10$

$$P(A \mid B) = \frac{0.20 \cdot 0.30}{0.20 \cdot\ 0.30 + 0.80 \cdot 0.10} = 0.4286$$

question (Monty Hall Problem)

Suppose you are a contestant on a game show. The game involves three doors.
Behind one of the doors is a car, and behind the other two doors are goats.
You pick a door, say Door 1, but before it's opened, the host, who knows what's behind each door, opens another door, say Door 3, revealing a goat. Now, the host offers you the opportunity to switch your choice to Door 2. Should you switch?

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A: The car is behind Door 1 (your initial choice).
B: The host opens Door 3 to reveal a goat.

The probability of the car being behind any specific door initially is $P(A) = \frac{1}{3}$.
Given that the car is behind Door 1, the probability that the host opens Door 3, revealing a goat is $P(B|A)=1$, because the host will always open a door with a goat behind it.
Given that the car is **not** behind Door 1, the probability that the host opens Door 3 (revealing a goat) is $P(B|\bar{A})=1$, because the host will always open a door with a goat behind it.

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Given that the car is **not** behind Door 1, the probability that the host opens Door 3 (revealing a goat) is $P(B|\bar{A})=1$, because the host will always open a door with a goat behind it.

$$P(A \mid B) = \frac{P(A)\, P(B \mid A)}{P(A)\, P(B \mid A) + P(\bar{A})\, P(B \mid \bar{A})} = \frac{\frac{1}{3} \cdot 1}{\frac{1}{3} \cdot 1 + \frac{2}{3} \cdot 1} = \frac{1}{3}$$

So the probability that the car is behind Door 1 is unaffected by the host opening Door 3.
But the car has to be behind either Door 1 or Door 2, so P(winning the car if you change) = $\frac{2}{3}$

question (Monty Hall Problem)