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$$P\left(\begin{array}{l}\text{genetic}\\\text{condition}\end{array}\right) = 0.05$$

FALSE NEGATIVE: $$P\left(\text{negative} \,\middle|\, \begin{array}{l}\text{genetic}\\\text{condition}\end{array}\right) = 0.08$$

FALSE POSITIVE: $$P\left(\text{positive} \,\middle|\, \begin{array}{l}\text{no}\\\text{genetic}\\\text{condition}\end{array}\right) = P\left(\overline{\text{negative}} \,\middle|\, \overline{\begin{array}{l}\text{genetic}\\\text{condition}\end{array}}\right) = 0.02$$

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$$P\left(\overline{\text{genetic condition}}\right) = 0.95$$

$$P\left(\text{genetic condition}\right) = 0.05$$

FALSE NEGATIVE: $P\left(\text{negative} \mid \text{genetic condition}\right) = 0.08$

FALSE POSITIVE: $P\left(\text{positive} \mid \text{no genetic condition}\right) = P\left(\text{positive} \mid \overline{\text{genetic condition}}\right) = 0.02$

$$P\left(\text{positive} \mid \text{genetic condition}\right) = 1 - P\left(\text{negative} \mid \text{genetic condition}\right) = 1 - 0.08 = 0.92$$

$$P\left(\text{genetic condition} \mid \text{positive}\right) = \frac{P\left(\text{genetic condition}\right) P\left(\text{positive} \mid \text{genetic condition}\right)}{P\left(\text{genetic condition}\right) P\left(\text{positive} \mid \text{genetic condition}\right) + P\left(\overline{\text{genetic condition}}\right) P\left(\text{positive} \mid \overline{\text{genetic condition}}\right)}$$

$$= \frac{0.05 \cdot 0.92}{(0.05 \cdot 0.92) + (0.95 \cdot 0.02)} \approx 0.7077$$

BASIC COUNTING RULE

If we are asked to choose one item from each of two separate categories where there are m items in the first category and n items in the second category, then the total number of available choices is $\mathbf{m \cdot n}$.

This is sometimes called the **multiplication rule for probabilities**.

Example

Suppose you have 5 different colored shirts (red, blue, green, yellow, and black) and 4 different colored pants
(orange, purple, gray, and brown) in your wardrobe.

You want to select one shirt and one pair of pants to wear for the day.

Total number of different outfits:
#(shirts)·#(pants)
$= 5 \cdot 4 = 20$

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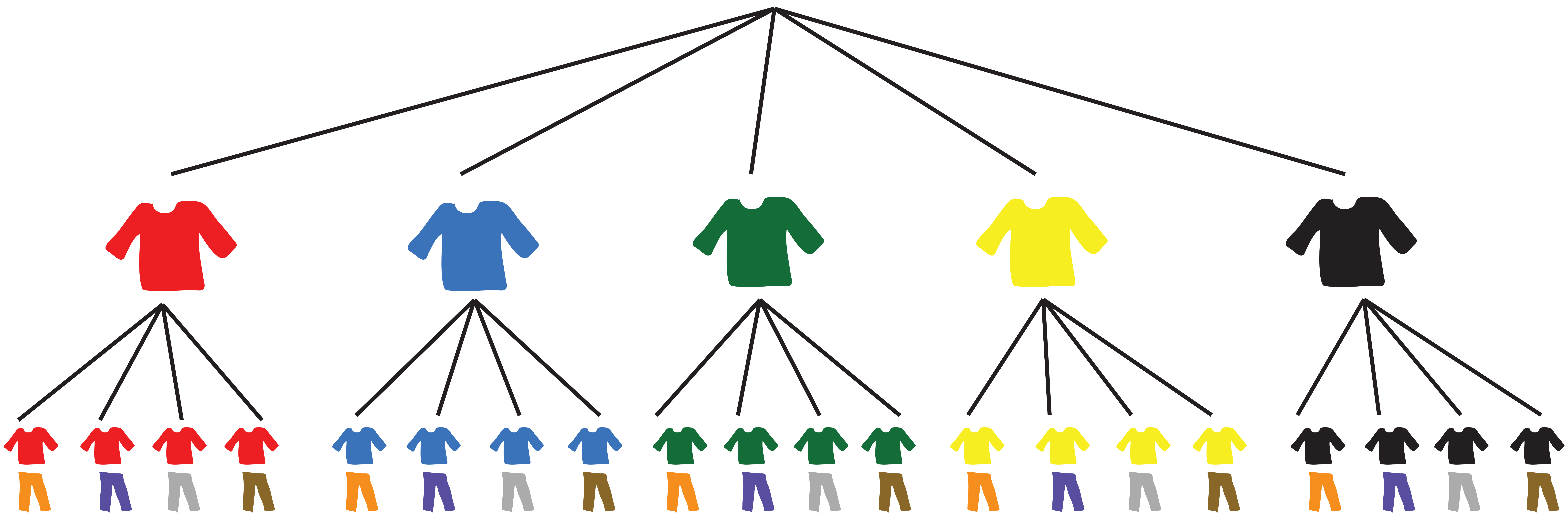

Example

Let's consider a scenario where you're organizing a sports event with different activities and teams.

Suppose you have the following options:

3 types of activities (football, basketball, volleyball)
4 teams (Team A, Team B, Team C, Team D)
2 time slots (morning, afternoon)

Total number of different combinations: #(options for activities) × #(options for teams) × #(options for time slots)

$$= 3 \cdot 4 \cdot 2 = 24$$

Therefore, there are 24 different combinations considering the type of activity, team, and time slot.

Question

You roll a die 3 three times. how many possibilities are there in total?

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Each roll has 6 possibilities.

total number of combinations $= 6 \cdot\ 6 \cdot\ 6 = 216$

question

You flip a coin 4 times. How many possible outcomes are there in total?

Question

How many combinations are there of a four digit numeric code?

Question

You draw a card from a standard deck of 52 cards 2 times, with replacement. How many possible outcomes are there in total?

Question

You spin a spinner with 8 equal sections 5 times. How many possible outcomes are there in total?

Question

how many different ways can we order the numbers

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad ?$$

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1 2 3 4 5 ?

1 2 3 4 5	2 1 3 4 5	3 1 2 4 5	4 1 2 3 5	5 1 2 3 4
1 2 3 5 4	2 1 3 5 4	3 1 2 5 4	4 1 2 5 3	5 1 2 4 3
1 2 4 3 5	2 1 4 3 5	3 1 4 2 5	4 1 3 2 5	5 1 3 2 4
1 2 4 5 3	2 1 4 5 3	3 1 4 5 2	4 1 3 5 2	5 1 3 4 2
1 2 5 3 4	2 1 5 3 4	3 1 5 2 4	4 1 5 2 3	5 1 4 2 3
1 2 5 4 3	2 1 5 4 3	3 1 5 4 2	4 1 5 3 2	5 1 4 3 2
1 3 2 4 5	2 3 1 4 5	3 2 1 4 5	4 2 1 3 5	5 2 1 3 4
1 3 2 5 4	2 3 1 5 4	3 2 1 5 4	4 2 1 5 3	5 2 1 4 3
1 3 4 2 5	2 3 4 1 5	3 2 4 1 5	4 2 3 1 5	5 2 3 1 4
1 3 4 5 2	2 3 4 5 1	3 2 4 5 1	4 2 3 5 1	5 2 3 4 1
1 3 5 2 4	2 3 5 1 4	3 2 5 1 4	4 2 5 1 3	5 2 4 1 3
1 3 5 4 2	2 3 5 4 1	3 2 5 4 1	4 2 5 3 1	5 2 4 3 1
1 4 2 3 5	2 4 1 3 5	3 4 1 2 5	4 3 1 2 5	5 3 1 2 4
1 4 2 5 3	2 4 1 5 3	3 4 1 5 2	4 3 1 5 2	5 3 1 4 2
1 4 3 2 5	2 4 3 1 5	3 4 2 1 5	4 3 2 1 5	5 3 2 1 4
1 4 3 5 2	2 4 3 5 1	3 4 2 5 1	4 3 2 5 1	5 3 2 4 1
1 4 5 2 3	2 4 5 1 3	3 4 5 1 2	4 3 5 1 2	5 3 4 1 2
1 4 5 3 2	2 4 5 3 1	3 4 5 2 1	4 3 5 2 1	5 3 4 2 1
1 5 2 3 4	2 5 1 3 4	3 5 1 2 4	4 5 1 2 3	5 4 1 2 3
1 5 2 4 3	2 5 1 4 3	3 5 1 4 2	4 5 1 3 2	5 4 1 3 2
1 5 3 2 4	2 5 3 1 4	3 5 2 1 4	4 5 2 1 3	5 4 2 1 3
1 5 3 4 2	2 5 3 4 1	3 5 2 4 1	4 5 2 3 1	5 4 2 3 1
1 5 4 2 3	2 5 4 1 3	3 5 4 1 2	4 5 3 1 2	5 4 3 1 2
1 5 4 3 2	2 5 4 3 1	3 5 4 2 1	4 5 3 2 1	5 4 3 2 1

FACTORIAL

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$$

Question

Suppose there are 5 different tasks (A, B, C, D, E) to be assigned to 5 employees (Alice, Bob, Charlie, David, Emma) in a company.

How many ways can the tasks be assigned to the employees?

Question

Suppose there are 7 different books to be placed on 7 different shelves in a library.

How many ways can the books be arranged on the shelves?

Question

Suppose there are 5 different seats in a row, and 5 friends need to sit in those seats.

How many ways can the friends be seated?

Question

Suppose there are 6 different colored balls (red, blue, green, yellow, purple, orange) to be placed into 6 distinct boxes.

How many ways can the balls be placed in the boxes?

question

In a deck of 52 playing cards, how many different ways can you draw three cards
in a specific order without replacement?

$$nPr = n \cdot (n-1) \cdot (n-2) \cdots (n-r+1)$$

We say that there are **nPr permutations of size r** that may be selected from among n choices without replacement when order matters.

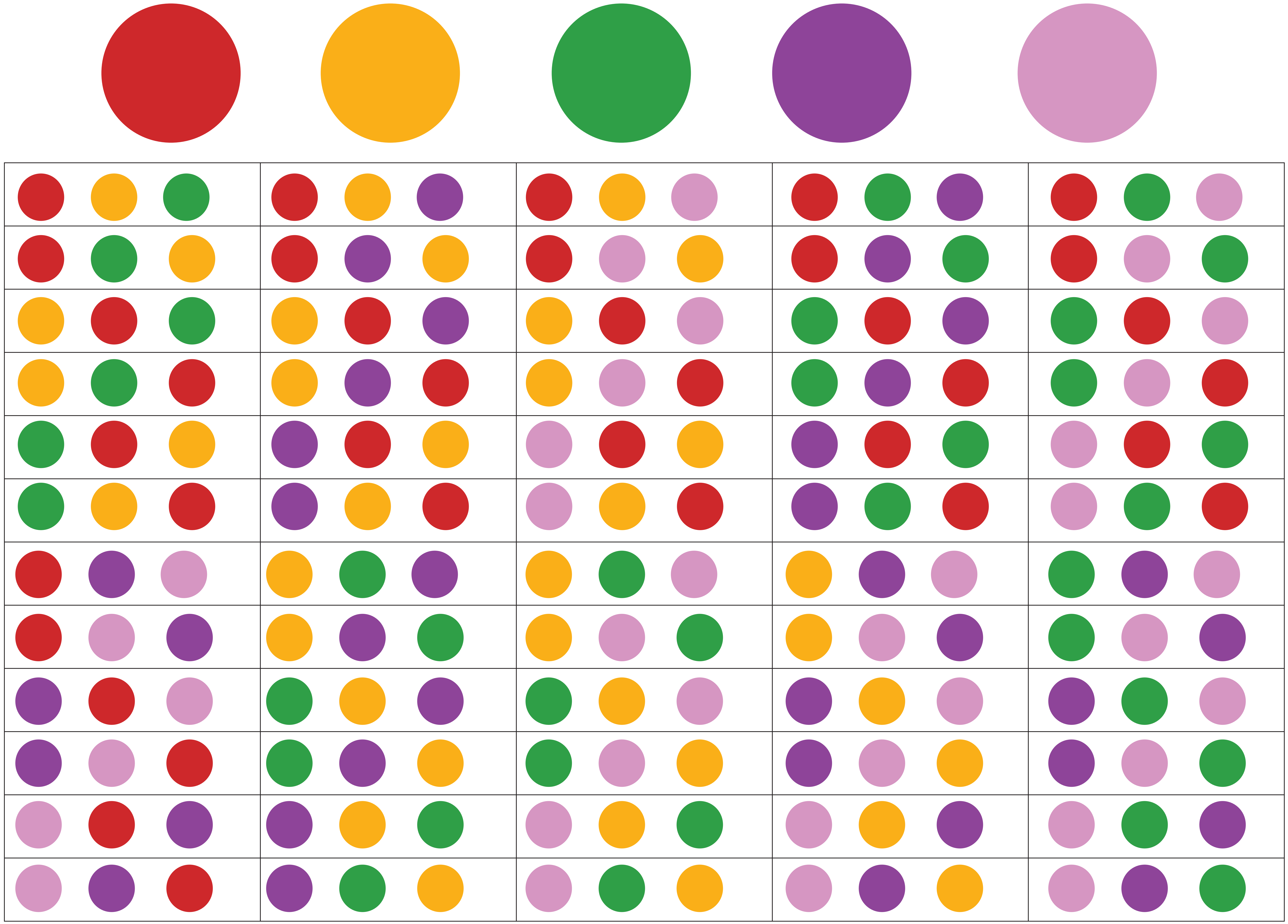
It turns out that we can express this result more simply using factorials.

$$nPr = \frac{n!}{(n-r)!}$$

Question

How many different ways can 3 students be chosen and arranged from a group of 5 students?

$$5P3 = \frac{5!}{(5\text{-}3)!} = \frac{5!}{2!} = \frac{5\times4\times3\times2\times1}{2\times1} = 60$$



Question

A restaurant has 7 different types of desserts, and the chef wants to create a special dessert platter with 3 different desserts. How many different ways can the chef arrange these desserts?

Question

A music festival features 9 different bands, and the organizer wants to schedule 4 of these bands to perform in a specific order. How many different ways can the organizer arrange these 4 bands?

Question

I have twelve different types of plants, and I want to arrange only five of them in a row on my garden bed. How many different ways could I do this?

COMBINATIONS

$$nCr = \frac{n!}{(n-r)!\ r!}$$

Question

How many ways can I choose 3 socks from a drawer containing 23 socks?

Question

In a group of 10 students, how many different ways can we choose a committee of 3 students to represent the class?

Question

A pizza restaurant offers 10 different toppings. How many different combinations of 5 toppings can a customer choose for their pizza?

Question

A sports team has 15 players, and the coach needs to select 4 players to represent the team in a tournament. How many different ways can the coach choose the players?

Birthday Paradox

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$$\underset{\approx 0.706}{1 - \frac{\frac{365!}{(365-30)!}}{365^{30}}}$$

Expected Value

Expected Value is the average gain or loss of an event if the procedure is repeated many times.

We can compute the expected value by multiplying each outcome by the probability of that outcome, then adding up the products.

Expected Value

$$E(X) = x_1 \cdot P(x_1) + x_2 \cdot P(x_2) + \cdots + x_n \cdot P(x_n) = \sum x \cdot P(x)$$

Example

You're participating in a local poker tournament with a buy-in of \$50. The tournament has 50 players. The prize pool is divided among the top 3 finishers, with the winner taking home \$1000, the second-place finisher receiving \$600, and the third-place finisher receiving \$300. Calculate the expected value of participating in this poker tournament.

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$X_1 = \$1000 - 50 = \$950$
$X_2 = \$600 - 50 = \$550$
$X_3 = \$300 - 50 = \$250$
$X_4 = -\$50$

$$E(X) = X_1 \cdot P(X_1) + X_2 \cdot P(X_2) + X_3 \cdot P(X_3) + X_4 \cdot P(X_4) =$$
$$= 950 \cdot \frac{1}{50} + 550 \cdot \frac{1}{50} + 250 \cdot \frac{1}{50} - 50 \cdot \frac{47}{50}$$
$$= 19 + 11 + 5 - 47 = -12$$

$P_1 = \frac{1}{50}$
$P_2 = \frac{1}{50}$
$P_3 = \frac{1}{50}$
$P_4 = \frac{47}{50}$

Example

A company is considering launching a new product. Market research indicates that there's a 60% chance of the product being successful and a 40% chance of it failing. If the product succeeds, the company expects to make a profit of $500,000. However, if the product fails, the company anticipates a loss of $200,000.
What is the expected value for the company in launching the new product?

Example

A company is considering launching a new product. Market research indicates that there's a 60% chance of the product being successful and a 40% chance of it failing. If the product succeeds, the company expects to make a profit of $500,000. However, if the product fails, the company anticipates a loss of $200,000.
What is the expected value for the company in launching the new product?

$P_{success} = 0.6$

$P_{failure} = 0.4$

$X_{success} = \$500{,}000$

$X_{failure} = -\$200{,}000$

$$E(X) = P_{success} \cdot X_{success} + P_{failure} \cdot X_{failure} = 0.6 \cdot 500{,}000 - 0.4 \cdot 200{,}000$$
$$= \$220{,}000$$