Payout Annuity Formula

$$P_0 = \frac{d(1-(1 + r/k)^{-Nk})}{(r/k)}$$

P_0 is the balance in the account at the beginning (starting amount, or principal).

d is the regular ***withdrawal*** (the amount you deposit each year, each month, etc.)

r is the annual interest rate in decimal form.

k is the number of compounding periods in one year.

When do you use this?

Payout annuities assume that you take money from the account on a regular schedule (every month, year, quarter, etc.) and let the rest sit there earning interest.

Compound interest: One deposit
Annuity: Many deposits.
Payout Annuity: Many withdrawals

Example

After retiring, you plan to withdraw $1,500 every month for 25 years from your retirement account. The account earns 5% interest annually, compounded monthly. How much money will you need in your account when you retire?

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After retiring, you plan to withdraw $1,500 every month for 25 years from your retirement account. The account earns 5% interest annually, compounded monthly. How much money will you need in your account when you retire?

d=1500: the monthly withdrawal

r=0.05: 5% annual interest rate

k=12: compounding occurs monthly

N=25: withdrawals are made for 25 years

$$P_0 = \frac{d(1-(1 + r/k)^{-Nk})}{(r/k)}$$

$$P_0 = \frac{1500(1-(1 + 0.05/12)^{-25 \cdot 12})}{(0.05/12)} = 279{,}495$$

Question

You plan to withdraw $2,000 every month for 15 years from your retirement account. The account earns 4% interest annually, compounded monthly. How much money will you need in your account when you retire?

Question

You want to withdraw $30,000 each year for 20 years. Your account earns 8% annual interest.

a) How much do you need in your account at the beginning?

Question

You want to withdraw $30,000 each year for 20 years. Your account earns 8% annual interest.

b) How much total money will you withdraw over the 20 years?

Question

You want to withdraw $30,000 each year for 20 years. Your account earns 8% annual interest.

c) How much of the withdrawn amount will come from interest?

Example

You know you will have $750,000 in your account when you retire. You want to take monthly withdrawals for a total of 25 years. Your retirement account earns 6% annual interest. How much will you be able to withdraw each month?

Example

You know you will have $750,000 in your account when you retire. You want to take monthly withdrawals for a total of 25 years. Your retirement account earns 6% annual interest. How much will you be able to withdraw each month?

r=0.06: 6% annual interest rate
k=12: compounding monthly
N=25: withdrawals for 25 years
P_0 =750,000: starting balance

$$P_0 = \frac{d(1-(1 + r/k)^{-Nk})}{(r/k)}$$

$$d= \frac{P_N (r/k)}{(1-(1 + r/k)^{-Nk})} = \frac{750{,}000\ (0.06/12)}{(1-(1 + 0.06/12)^{-25 \cdot 12})} = 4827.84$$

Question

You know you will have $400,000 in your account when you retire. You want to take monthly withdrawals for a total of 20 years. Your retirement account earns 7% annual interest. How much will you be able to withdraw each month?

Question

A donor contributes $250,000 to a hospital, with instructions that it should fund annual grants for the next 25 years. If the hospital can earn 5% annual interest, how much can they allocate for grants each year?

Loans Formula

$$P_0 = \frac{d(1-(1 + r/k)^{-Nk})}{(r/k)}$$

P_0 is the balance in the account at the beginning (starting amount, or principal).

d is the loan payment (the amount you pay each year, each month, etc.)

r is the annual interest rate in decimal form.

k is the number of compounding periods in one year.

N is the length of the loan in years.

When do you use this?

The loan formula assumes that you make loan payments on a regular schedule (every month, year, quarter, etc.) and are paying interest on the loan.

Compound interest: One deposit
Annuity: Many deposits
Payout Annuity: Many withdrawals
Loans: Many payments

Example

You can afford $300 per month as a car payment. If you can get an auto loan at 4% interest for 72 months (6 years), how expensive of a car can you afford? In other words, what loan amount can you pay off with $300 per month?

Example

You can afford $300 per month as a car payment. If you can get an auto loan at 4% interest for 72 months (6 years), how expensive of a car can you afford? In other words, what loan amount can you pay off with $300 per month?

d=300: the monthly loan payment
r=0.04: 4% annual interest rate
k=12: monthly compounding
N=6: payments for 6 years (72 months)

$$P_0 = \frac{d(1-(1 + r/k)^{-Nk})}{(r/k)}$$

$$P_0 = \frac{300(1-(1 + 0.04/12)^{-6 \cdot 12})}{(0.04/12)} = 19{,}098$$

Question

You can afford $250 per month as a car payment. If you secure an auto loan at 5% interest for 48 months (4 years), how expensive of a car can you afford? In other words, what loan amount can you pay off with $250 per month?

Question

You want to take out a $200,000 mortgage (home loan). The interest rate on the loan is 4%, and the loan is for 15 years. How much will your monthly payments be?

Question

If a mortgage at a 5% interest rate has payments of $1,500 per month and the loan term is 30 years, how much will the loan balance be 5 years from the end of the loan?

Example

Maria is considering putting a $2,000 phone purchase on her credit card, which has an interest rate of 15% compounded monthly. How long will it take her to pay off the purchase if she makes monthly payments of $50?

Example

Maria is considering putting a $2,000 phone purchase on her credit card, which has an interest rate of 15% compounded monthly. How long will it take her to pay off the purchase if she makes monthly payments of $50?

d=50: monthly payments
r=0.15: 15% annual interest rate
k=12: monthly compounding
P_0 =2,000: initial loan amount

$$P_0 = \frac{d(1-(1 + r/k)^{-Nk})}{(r/k)}$$