Annuity Formula

$$P_N = \frac{d((1 + r/k)^{Nk} - 1)}{(r/k)}$$

P_N is the balance in the account after N years. d is the regular deposit (the amount you deposit each year, each month, etc.)

r is the annual interest rate in decimal form.

k is the number of compounding periods in one year.

If the compounding frequency is not explicitly stated, assume there are the same number of compounds in a year as there are deposits made in a year.

When do you use this?

Annuities assume that you put money in the account on a regular schedule (every month, year, quarter, etc.) and let it sit there earning interest.

Compound interest assumes that you put money in the account once and let it sit there earning interest.

Compound interest: One deposit
Annuity: Many deposits.

Example

A savings plan allows you to deposit money monthly into an account that earns interest. Suppose you deposit $150 each month into an account earning 5% annual interest, compounded monthly. How much will you have saved after 25 years?

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Monthly deposit d = $150
Annual interest rate (r): 5% (r = 0.05 as a decimal)
Compounding periods per year k=12 (compounded monthly)
Number of years N = 25

Example

A savings plan allows you to deposit money monthly into an account that earns interest. Suppose you deposit $150 each month into an account earning 5% annual interest, compounded monthly. How much will you have saved after 25 years?

Monthly deposit **d** = **$150**
Annual interest rate (r): 5% (**r** = **0.05** as a decimal)
Compounding periods per year **k=12** (compounded monthly)
Number of years **N** = **25**

$$P_N = \frac{d\left(\left(1 + r/k\right)^{Nk} - 1\right)}{(r/k)} \qquad P_N = \frac{150\left(\left(1 + 0.05/12\right)^{25\text{x}12} - 1\right)}{(0.05/12)}$$

$$= \$162{,}561$$

Question

You decide to save for a down payment on a house by depositing \$200 each month into a savings account that earns 4% annual interest, compounded monthly. How much will you have saved after 15 years?

Example

A savings account pays 4% interest. If you deposit \$10 a day into this account, how much will you have after 15 years? How much of that amount is from interest?

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A savings account pays 4% interest. If you deposit $10 a day into this account, how much will you have after 15 years? How much of that amount is from interest?

Daily deposit d= $10

Annual interest rate (r): 4% (r = 0.04 as a decimal)

Compounding periods per year k=365 (compounded daily)

Number of years N = 15

$$P_N = \frac{d((1 + r/k)^{Nk} - 1)}{(r/k)}$$

$$P_N = \frac{10((1 + 0.04/365)^{15\text{x}365} - 1)}{(0.04/365)}$$

$$= \$92{,}370$$

Total deposits: 10 × 365 × 15 = $54,750

Interest earned: $92,370 - $54,750 = $37,620

Question

A retirement savings account offers 2.5% annual interest. If you deposit $3 per day into this account, how much will you have after 8 years? How much of that total will come from interest?

Question

You decide to invest $200 each month into an account earning 5% annual interest, compounded monthly.

a) How much will you have in the account after 25 years?

b) How much total money will you contribute to the account?

c) How much of the total balance will come from interest?

Example

You want to save \$150,000 for a down payment on a house in 20 years. Your savings account earns 6% annual interest, compounded monthly. How much do you need to deposit each month to reach your goal?

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You want to save $150,000 for a down payment on a house in 20 years. Your savings account earns 6% annual interest, compounded monthly. How much do you need to deposit each month to reach your goal?

Annual interest rate (r): 6% (r = 0.06 as a decimal)
Compounding periods per year k=12 (monthly deposits)
Number of years N=20
Target amount P_{20}= $150,000

Example

You want to save \$150,000 for a down payment on a house in 20 years. Your savings account earns 6% annual interest, compounded monthly. How much do you need to deposit each month to reach your goal?

Annual interest rate (r): 6% (r = 0.06 as a decimal)
Compounding periods per year k=12 (monthly deposits)
Number of years N=20
Target amount P_{20}= \$150,000

$$P_N = \frac{d((1 + r/k)^{Nk} - 1)}{(r/k)}$$

$$d = \frac{P_N (r/k)}{((1 + r/k)^{Nk} - 1)} = \frac{150{,}000 \times (0.06/12)}{((1+0.06/12)^{20\times12} - 1)} \approx \$324.68$$

Question

You want to save $250,000 for your child's college education in 18 years. Your investment account earns 7% annual interest, compounded monthly. How much do you need to deposit each month to reach your goal?

Example

If you invest \$50 each month into an account earning 4% annual interest, compounded monthly, how long will it take for the account to grow to \$5,000?

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If you invest \$50 each month into an account earning 4% annual interest, compounded monthly, how long will it take for the account to grow to \$5,000?

Monthly deposit d= \$50

Annual interest rate (r): 4% (r=0.04 as a decimal)

Compounding periods per year k=12 (monthly deposits)

Target amount P_N = \$5,000

Unknown (N): Time in years

Example

If you invest \$50 each month into an account earning 4% annual interest, compounded monthly, how long will it take for the account to grow to \$5,000?

Monthly deposit d= \$50

Annual interest rate (r): 4% (r=0.04 as a decimal)

Compounding periods per year k=12 (monthly deposits)

Target amount P_N = \$5,000

Unknown (N): Time in years

$$P_N = \frac{d((1 + r/k)^{Nk} - 1)}{(r/k)} \qquad \frac{P_N (r/k)}{d} + 1 = (1 + r/k)^{Nk}$$

$$\frac{P_N (r/k)}{d} = ((1 + r/k)^{Nk} - 1) \qquad \log\left[\frac{P_N (r/k)}{d} + 1\right] = \log[(1 + r/k)^{Nk}] = Nk \log(1 + r/k)$$

$$N = \frac{\log\left[\frac{P_N (r/k)}{d} + 1\right]}{k \log(1 + r/k)} = \frac{\log\left[\frac{5000(0.04/12)}{50} + 1\right]}{12 \log(1 + 0.04/12)} \approx 7.2$$