Compound Interest

$P_n = P_0(1 + r/k)^{Nk}$

P_N is the balance in the account after N years.
P_0 is the starting balance of the account (also called initial deposit, or principal)

r is the annual interest rate in decimal form
k is the number of compounding periods in one year
If the compounding is done annually (once a year), k = 1.
If the compounding is done quarterly, k = 4.
If the compounding is done monthly, k = 12.
If the compounding is done daily, k = 365.

Example

A new savings bond offers a fixed interest rate and compounds quarterly, providing an attractive option for long-term investments. Suppose you invest $5,000 in a savings bond with an annual interest rate of 4%, compounded quarterly. How much will your investment be worth after 15 years?

Example

A new savings bond offers a fixed interest rate and compounds quarterly, providing an attractive option for long-term investments. Suppose you invest $5,000 in a savings bond with an annual interest rate of 4%, compounded quarterly. How much will your investment be worth after 15 years?

P_0 =5000 (initial deposit)

$P_n=P_0(1+r/k)^{Nk}$

r = 0.04 (4% annual rate)}

k= 4 (4 quarters in a year)

N=15 (15 years)

The future value formula is:

$P_N =P_0(1+ k/r)^{N\cdot k}$

Substituting the values: $P_{15} = 5000(1+ 4/0.04)^{15\cdot 4} = \$9024.80$

Question

An investment account offers an annual interest rate of 5%, compounded semiannually, to encourage long-term savings. Suppose you deposit $7,500 into the account. How much will the account balance be after 10 years?

Question

A retirement fund offers an annual interest rate of 4.5%, compounded annually, to help investors grow their savings. Suppose you contribute $10,000 to this fund. How much will the account be worth after 12 years?

Example

You know that you will need $25,000 for a down payment on a house in 15 years. If your account earns 3.5% interest compounded monthly, how much would you need to deposit now to reach your goal?

Example

You know that you will need $25,000 for a down payment on a house in 15 years. If your account earns 3.5% interest compounded monthly, how much would you need to deposit now to reach your goal?

$$P_n = P_0(1+r/k)^{Nk}$$

$$P_0 = \frac{P_N}{(1+r/k)^{N \cdot k}}$$

$$P_0 = \frac{25{,}000}{(1+0.035/12)^{15 \cdot 12}} = \$15{,}161.35$$

Question

You want to save $30,000 for a car purchase in 5 years. If your savings account earns 6% interest compounded semiannually, how much do you need to deposit now to meet your goal?

Example

If you invest $5,000 at 5% interest compounded quarterly, how long will it take for the account to triple in value?

Example

If you invest $5,000 at 5% interest compounded quarterly, how long will it take for the account to triple in value?

$P_0 = 5000$ (initial investment)
$P_N = 3 \times P\ 0 = 15{,}000$ (tripled value)
$r = 0.05$ (5% annual interest rate)
$k = 4$ (4 compounding periods per year)

The formula is: $P_N = P_0(1 + k/r)^{N \cdot k}$

Rearranging to solve for $N \cdot k = \log(P_N/P_0) / \log(1 + r/k)$
Substitute the values: $N \cdot 4 = \log(15{,}000/5{,}000)/\log(1 + 0.05/4)$
$N \cdot 4 = \log(3)/\log(1.0125)$
$N \cdot 4 = 0.4771/0.0054 = 88.35$
$N = 88.35/4 = 22.09$ years

Question

If you invest $3,000 at 4% interest compounded annually, how long will it take for the account to grow to $5,000?