COMMON LOGARITHM

The **common logarithm**, written log(x), undoes the exponential 10^x

This means that $\log(10^x) = x$, and likewise $10^{\log(x)} = x$

This also means the statement $10^a = b$ is equivalent to the statement $\log(b) = a$

log(x) is read as "log of x", and means "the logarithm of the value x". It is important to note that this is not multiplication – the log doesn't mean anything by itself, just like √ doesn't mean anything by itself; it has to be applied to a number.

Example

log(10) = 1

log(100) = 2

log(1,000) = 3

log(10,000) = 4

log(1) = 0

log(1/10) = -1

log(1/100) = -2

log(1/1,000) = -3

Question

Evaluate log(50) (use a calculator).

Question

Solve $10^x = 100$

Question

Solve $10^x = 56$

Question

Solve $3(10^x) = 5$

Properties of Logs: Exponent Property

$$\log(A^r) = r\log(A)$$

Properties of Logs: Exponent Property

$$\log(A^r)=r\log(A)$$

$$A = 10^{\log A}$$

$$A^r = (10^{\log A})^r$$

$$A^r = 10^{r \log A}$$

$$\log(A^r) = \log(10^{r \log A})$$

$$\log(A^r) = r \log A$$

Example

Rewrite log(36) using the exponent property for logs.

Example

Rewrite $\log(36)$ using the exponent property for logs.

$$\log(36) = \log(6^2) = 2\log(6)$$

Question

Rewrite log(27) using the exponent property for logs.

Solving exponential equations with logarithms

1. Isolate the exponential. In other words, get it by itself on one side of the equation. This usually involves dividing by a number multiplying it.

2. Take the log of both sides of the equation.

3. Use the exponent property of logs to rewrite the exponential with the variable exponent multiplying the logarithm.

4. Divide as needed to solve for the variable.

Example

The town of Crestwood has been expanding its park area according to the equation $P_n = 320(1.04)^n$, where P_n represents the years after 2015, and the area P_n is measured in acres. In which year will Crestwood's park area reach 500 acres?

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$P_n = 320(1.04)^n$, where n represents the years after 2015, and the area P_n is measured in acres. In which year will Crestwood's park area reach 500 acres?

$$500 = 320(1.04)^n \qquad \frac{500}{320} = (1.04)^n$$

$1.5625 = (1.04)^n$
$\log(1.5625) = \log((1.04)^n)$

$\log(1.5625) = n \log(1.04)$
$n = \log(1.5625)/\log(1.04) \approx 11.38$

Question

A city's annual budget for public transportation has been growing according to the equation $Pn = 150(1.05)^n$, where n is the number of years after 2010, and the budget P_n is measured in millions of dollars. In what year will the budget reach 300 million dollars?

Question

An art restoration process involves applying protective coatings to an ancient painting to reduce fading. Each layer of coating blocks 80% of the remaining light exposure. If the painting originally received 200,000 lux of light exposure per month, how many layers are needed to reduce the light exposure to below 1,000 lux per month?

Question

A lake is treated to reduce invasive algae by adding a special chemical that removes 70% of the remaining algae with each application. If the lake starts with 5 million algae cells per liter, how many treatments are needed to bring the algae count down to 2,000 cells per liter?

CARRYING CAPACITY

The **carrying capacity**, or maximum sustainable population, is the largest population that an environment can support.

CARRYING CAPACITY

If a population is growing in a constrained environment with carrying capacity K, and absent constraint would grow exponentially with growth rate r, then the population behavior can be described by the logistic growth model:

$$P_n = P_{n-1} + r\left(1 - \frac{P_{n-1}}{K}\right)P_{n-1}$$

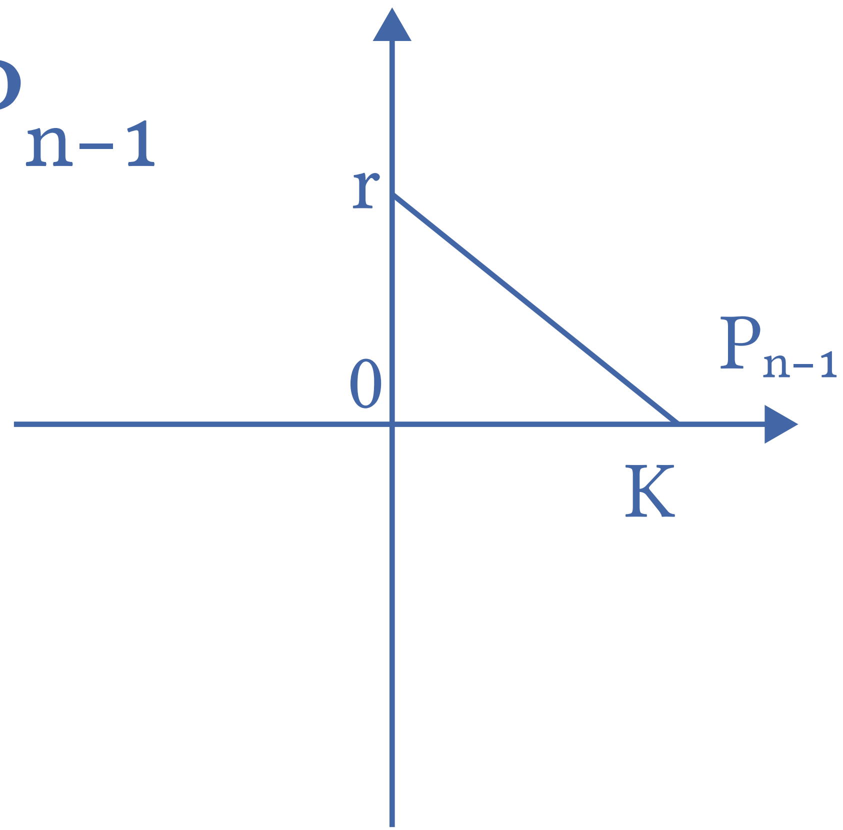
EXPONENTIAL GROWTH

$$P_n = (1 + r)\, P_{n-1}$$

$$P_n = P_{n-1} + r\, P_{n-1}$$

LOGISTIC GROWTH

$$P_n = P_{n-1} + r\left(1 - \frac{P_{n-1}}{K}\right)P_{n-1}$$



Example

An isolated island currently has a population of 300 wild goats. Ecologists estimate the island's ecosystem can support a maximum of 3,000 goats. Without any external factors, the goat population would increase by 40% per year. Predict the future population using the logistic growth model.

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$$P_1 = P_0 + 0.40\left(1 - \frac{P_0}{3000}\right)P_0 = 300 + 0.40\left(1 - \frac{300}{3000}\right) 300 = 408$$

Using this to calculate the following year:

$$P_2 = P_1 + 0.40\left(1 - \frac{P_1}{3000}\right)P_1 \approx 408 + 0.40\left(1 - \frac{408}{3000}\right) 408 \approx 559$$

Question

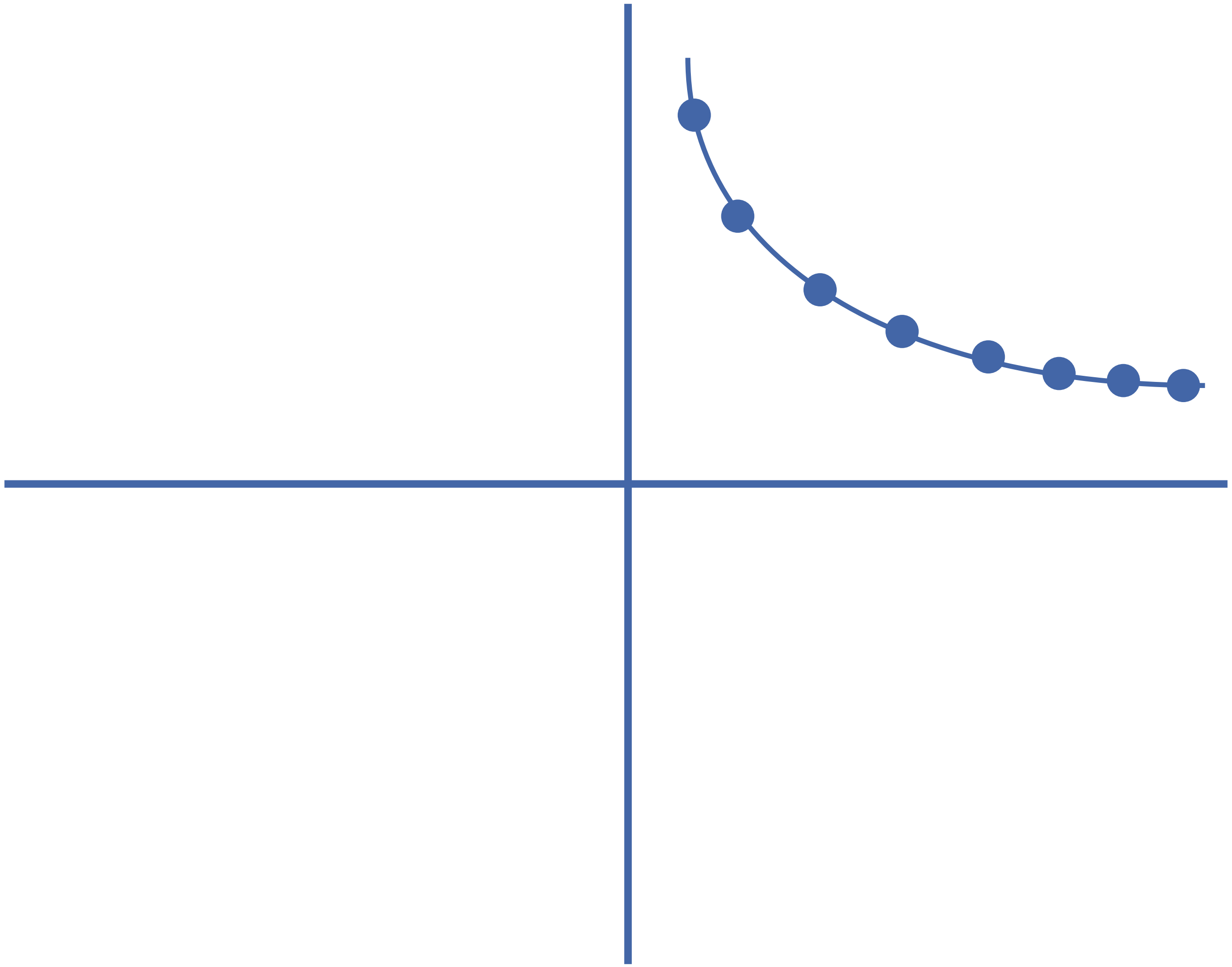
A fish tank currently houses 30 goldfish. Without any limitations, the number of goldfish would increase by 80% each year, but the tank can only support a maximum of 250 fish. Use the logistic growth model to predict the fish population in the next three years.

Question

A wildlife reserve currently has 45 deer. If left unregulated, the population would grow by 65% each year, but the reserve can only support a maximum population of 500 deer. Use the logistic growth model to predict the deer population in the next three years.

Question

What growth best represents the following graph?



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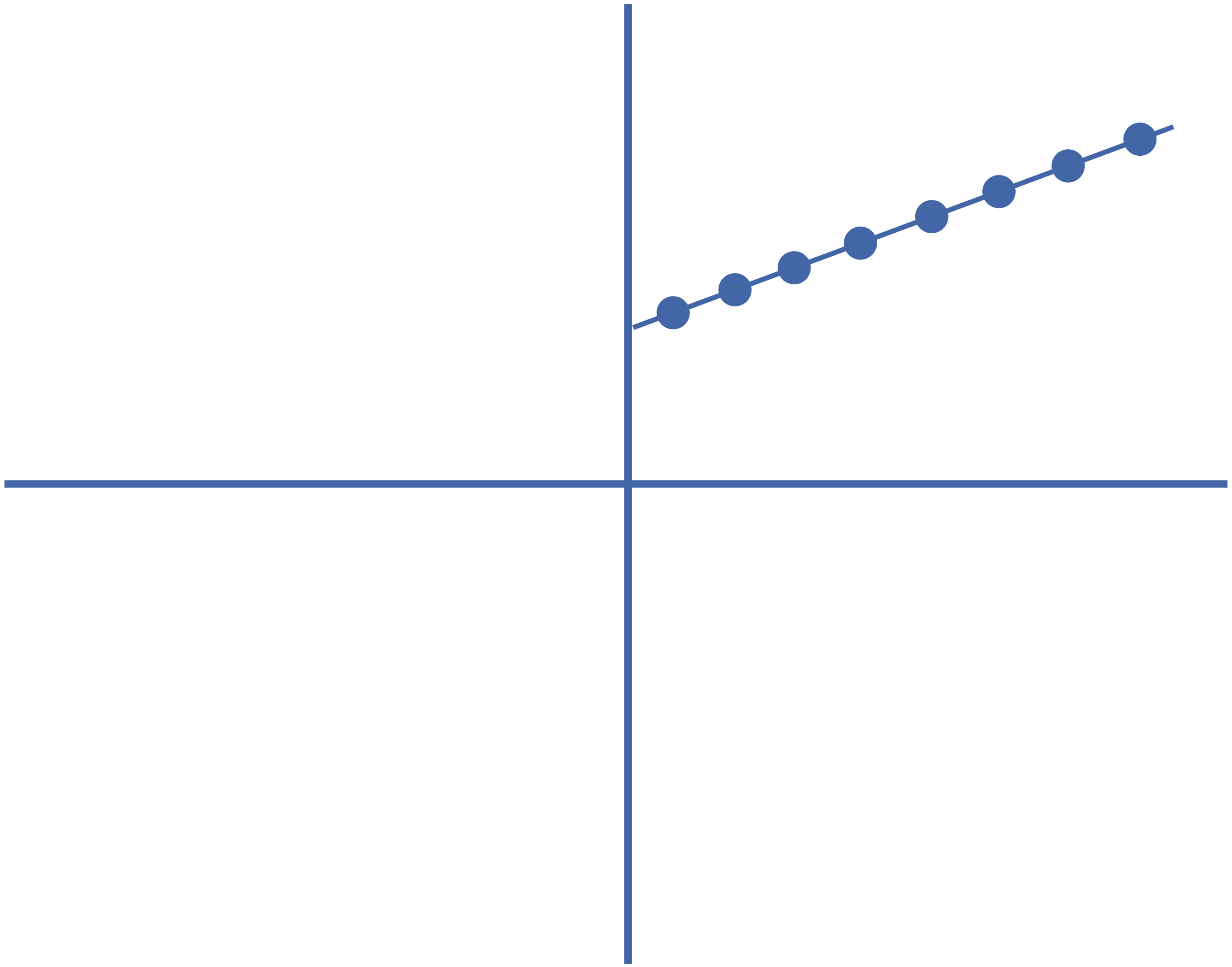

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