LINEAR GROWTH

If a quantity starts at size P_0 and grows by d every time period, then the quantity after n time periods can be determined using either of these relations:

Recursive form

$$P_n = P_{n-1} + d$$

Explicit form

$$P_n = P_0 + d \cdot n$$

In this equation, d represents the common difference – the amount that the population changes each time n increases by 1.

$$P_n = P_0 + d \cdot n \qquad y = m \cdot x + c$$

Example

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P0 = 450 P6 = 630

$$d = \text{slope} = \frac{P6 - P0}{6 - 0} = \frac{630}{6} = 105$$

$P_n = P_0 + d \cdot n$, $P_n = 450 + 105d$ EXPLICIT

$P_0 = 450$, $P_n = P_{n-1} + 105$ RECURSIVE

Question

In a protected park, the number of oak trees was recorded at 3,200 in 2015. By 2020, the oak tree population had increased to 4,000. Assuming the tree population grows at a constant rate, what is the expected oak tree population in 2028?

Question

A research station observed a penguin colony population of 1,500 in 2011, which grew to 1,950 by 2016. If the penguin population continues to grow at this same rate, what will the population be in 2023?

Example

The water consumption in a growing city has been increasing steadily. The data for water usage (in billions of gallons) from 2005 to 2017 is shown below. Find a model for this data and use it to predict the water usage in 2025. If the trend continues, in what year will the water usage reach 220 billion gallons?

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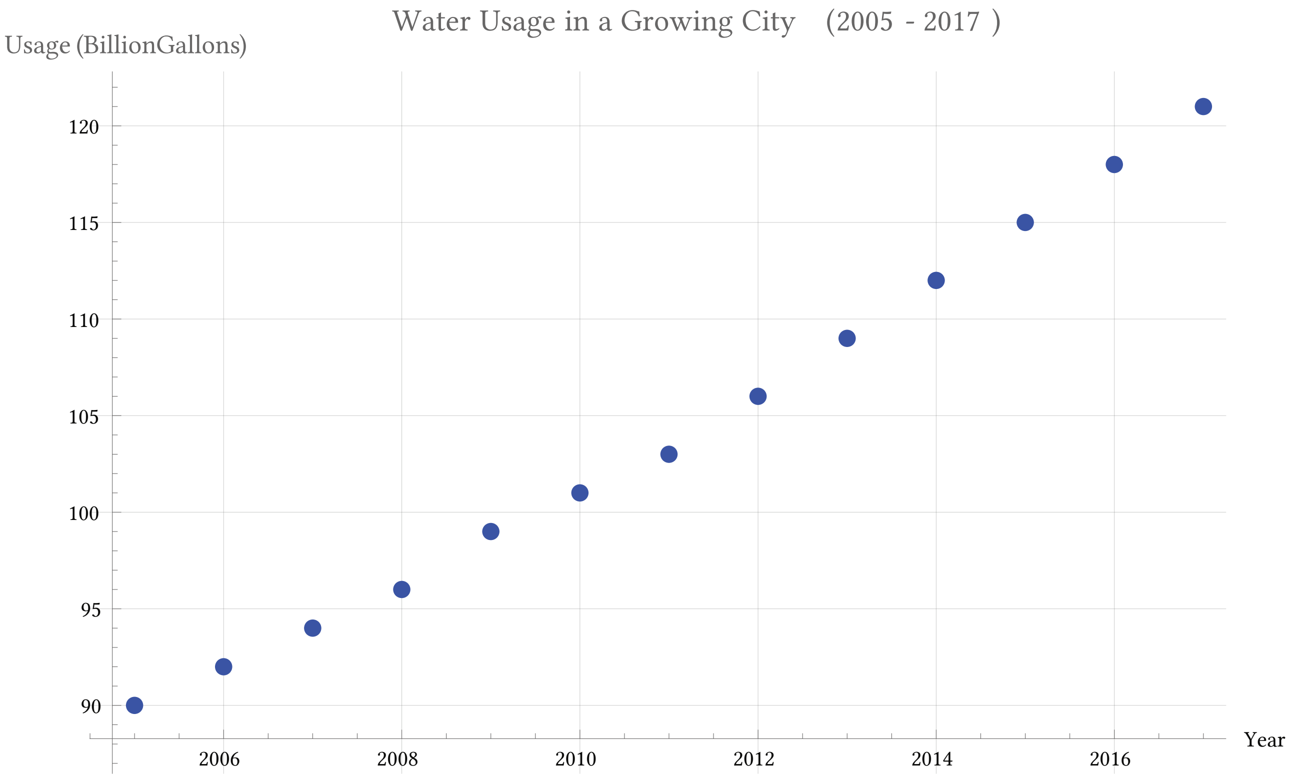

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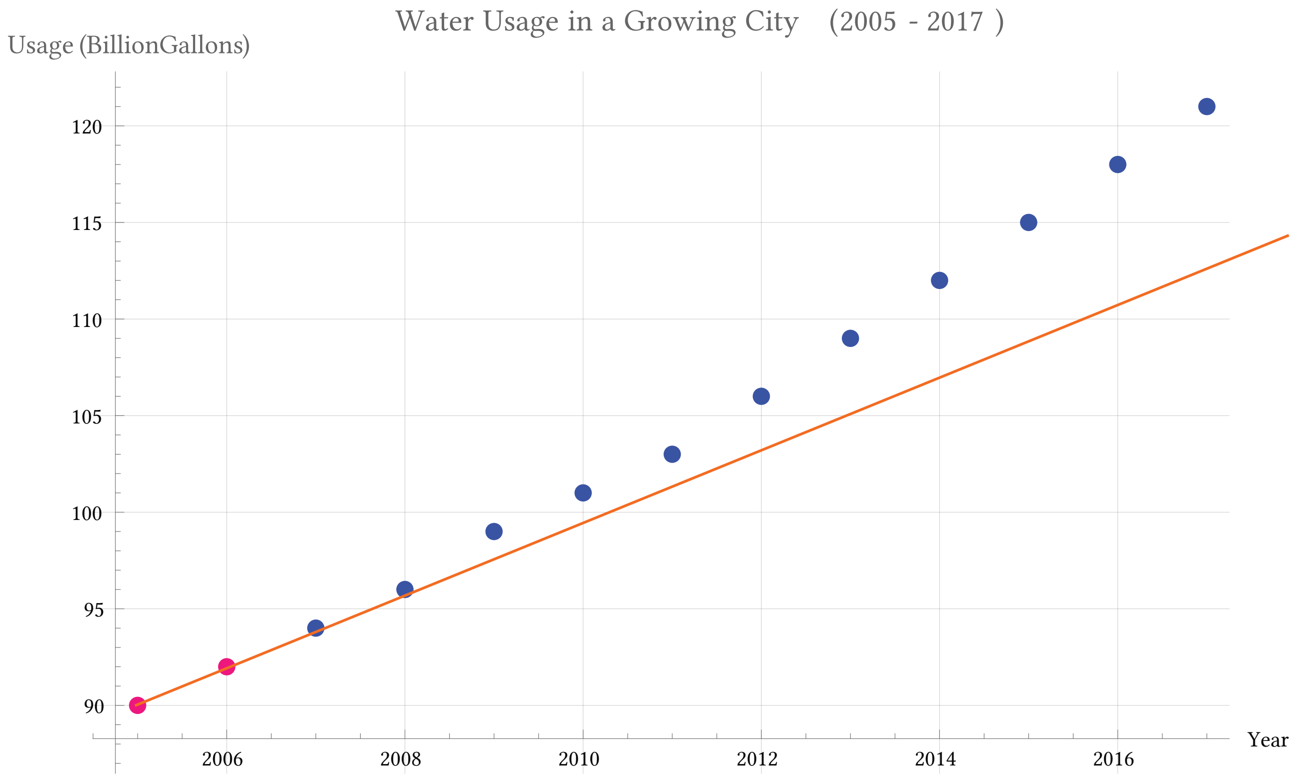

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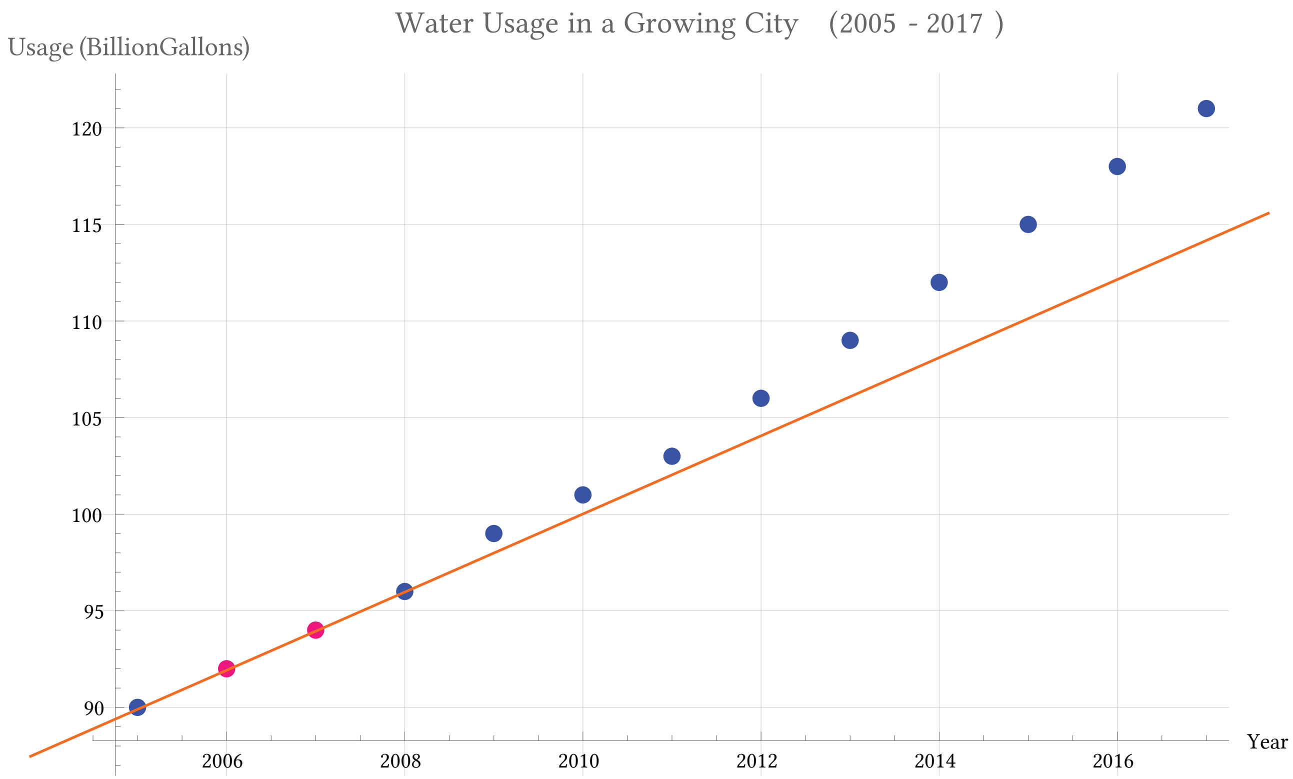

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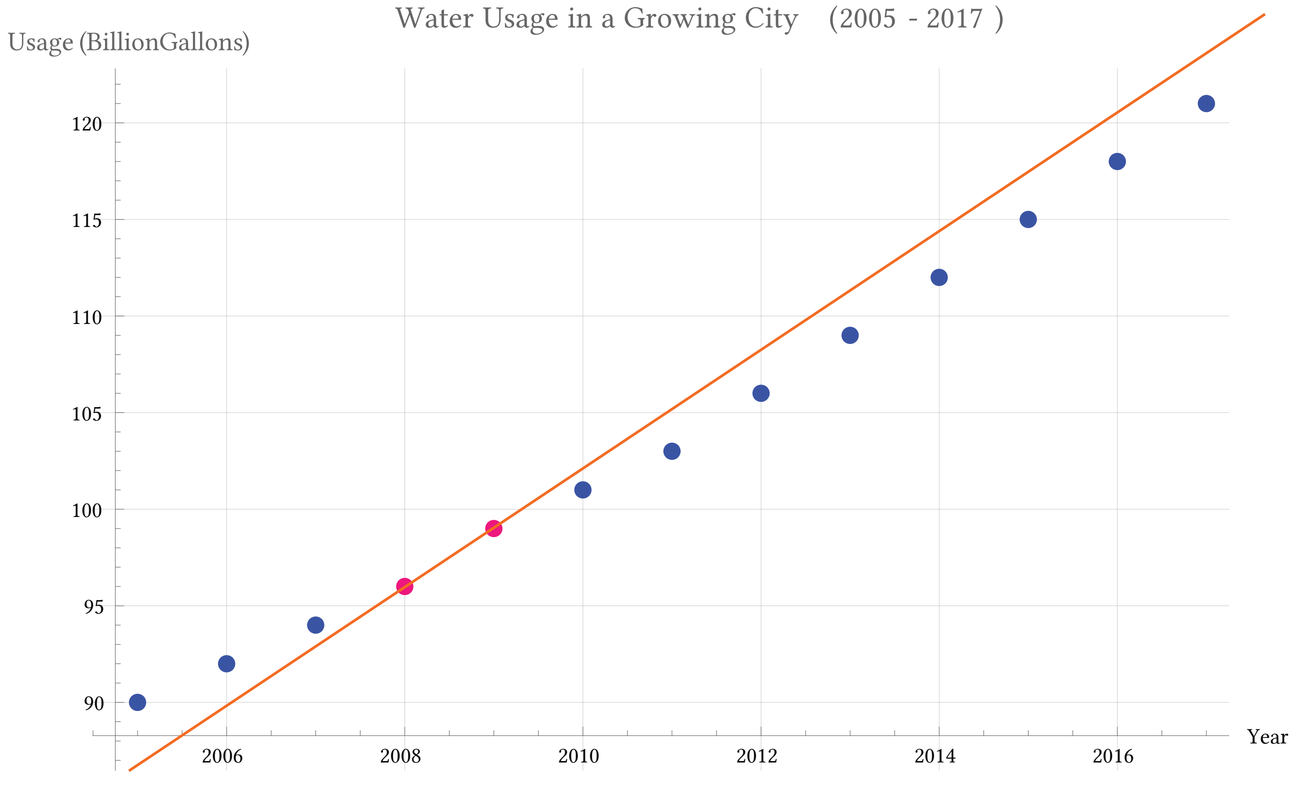

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The value for C_0 in this equation is 500, so the initial starting cost is $500. This suggests there is an initial setup or administrative fee of $500 to start renting the unit. The value for d in the equation is 45, which means the cost increases by $45 each month. This tells us that the monthly rental fee for the storage unit is $45 per month.

Question

The number of women working in STEM fields in a certain country has been increasing over recent decades. Although the growth isn't perfectly linear, it is fairly consistent. Use the data from 1985 and 2015 to find an explicit formula for the number of women in STEM, then use it to estimate the number in 2025.

Year	1985	1990	2000	2010	2015
# of Women in STEM	15,000	18,250	26,500	34,700	39,800

Question

Consider a population of a species of birds that grows according to the recursive rule $P_n = P_{n-1} + 120$, with an initial population of $P_0 = 200$. Then:

$P_1 =$ $\qquad$ P2 =

Find an explicit formula for the population. Your formula should involve n (use lowercasen).

$P_n =$

Use your explicit formula to find P_{50}.

$P_{50} =$

Question

A young tree is currently 5 feet tall, and it is expected to grow 1.5 feet each year. Create a linear growth model, with n=0 representing the current height of the tree.

EXPONENTIAL GROWTH

If a quantity starts at size P_0 and grows by R% (written as a decimal, r) every time period, then the quantity after n time periods can be determined using either of these relations:

Recursive form: $P_n = (1+r) \cdot P_{n-1}$

Explicit form: $P_n = (1+r)^n \cdot P_0$ or $P_n = P_0(1+r)^n$

We call r the **growth rate**.

The term (1+r) is called the **growth multiplier**, or **common ratio**.

Example

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First, we need to define the year corresponding to n=0.
Since we know the population in 2016, it makes sense to let 2016 correspond to n=0, so $P_0 = 12{,}500$. The year 2022 would then be n=6. The growth rate is 4%, giving r=0.04.

Using the explicit form:
$P_6 = (1+0.04)^6 \cdot 12{,}500 = (1.265319) \cdot 12{,}500 \approx 15{,}816.49$

The model predicts that in 2022, the town would have a population of about 15,816 people.

Question

In a recent fiscal year, a local coffee shop reported revenues of $200,000, reflecting a growth of about 10% from the previous year. If this growth rate continues, what would the revenue of the coffee shop be in 2025?

Evaluating exponents on the calculator

To evaluate expressions like $(1.03)^6$, it will be easier to use a calculator than multiply 1.03 by itself six times. Most scientific calculators have a button for exponents. It is typically either labeled like:

^ , yx , or xy .

To evaluate 1.03^6 we'd type 1.03 ^ 6, or 1.03 y^x 6.

Try it out – you should get an answer around 1.1940523.

Question

Brazil is the fifth largest country in the world by area, with a population in 2024 of approximately 212 million people. The population is growing at a rate of about 0.4% each year. If this trend continues, what will Brazil's population be in 2050?

ROUNDING

A very small difference in the growth rates gets magnified greatly in exponential growth. For this reason, it is recommended to round the growth rate as little as possible.

If you need to round, keep at least three significant digits – numbers after any leading zeros.   So 0.4162 could be reasonably rounded to 0.416. A growth rate of 0.001027 could be reasonably rounded to 0.00103.

Question

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