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$$1 - \frac{\frac{365!}{(365-30)!}}{365^{30}}$$

≈ 0.706

Expected Value

Expected Value is the average gain or loss of an event if the procedure is repeated many times.

We can compute the expected value by multiplying each outcome by the probability of that outcome, then adding up the products.

Expected Value

$$E(X) = x_1 \cdot P(x_1) + x_2 \cdot P(x_2) + \cdots + x_n \cdot P(x_n) = \sum x \cdot P(x)$$

Example

You're participating in a local poker tournament with a buy-in of $50. The tournament has 50 players. The prize pool is divided among the top 3 finishers, with the winner taking home $1000, the second-place finisher receiving $600, and the third-place finisher receiving $300. Calculate the expected value of participating in this poker tournament.

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X_1 = $1000 - 50 = $950
X_2 = $600 - 50 = $550
X_3 = $300 - 50 = $250
X_4 = -$50

$$E(X) = X_1 \cdot P(X_1) + X_2 \cdot P(X_2) + X_3 \cdot P(X_3) + X_4 \cdot P(X_4) =$$
$$= 950 \cdot \frac{1}{50} + 550 \cdot \frac{1}{50} + 250 \cdot \frac{1}{50} - 50 \cdot \frac{47}{50} = -9$$

$P_1 = \frac{1}{50}$
$P_2 = \frac{1}{50}$
$P_3 = \frac{1}{50}$
$P_4 = \frac{47}{50}$

Example

A company is considering launching a new product. Market research indicates that there's a 60% chance of the product being successful and a 40% chance of it failing. If the product succeeds, the company expects to make a profit of $500,000. However, if the product fails, the company anticipates a loss of $200,000.
What is the expected value for the company in launching the new product?

Example

A company is considering launching a new product. Market research indicates that there's a 60% chance of the product being successful and a 40% chance of it failing. If the product succeeds, the company expects to make a profit of $500,000. However, if the product fails, the company anticipates a loss of $200,000.
What is the expected value for the company in launching the new product?

$P_{success} = 0.6$

$P_{failure} = 0.4$

$X_{success} = \$500,000$
$X_{failure} = -\$200,000$

$$E(X) = P_{success} \cdot X_{success} + P_{failure} \cdot X_{failure} = 0.6 \cdot 500,000 - 0.4 \cdot 200,000$$
$$= \$220,000$$