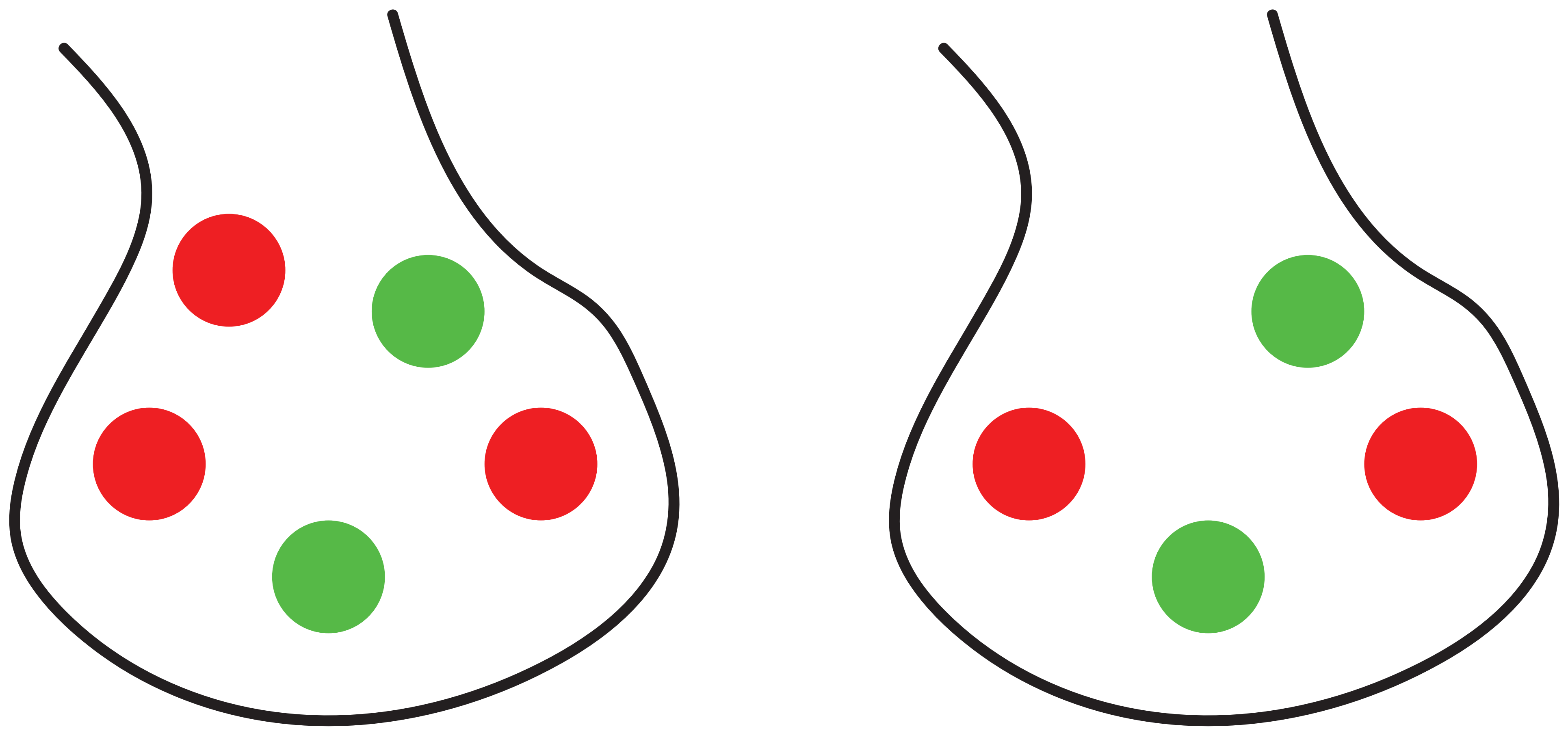
CONDITIONAL PROBABILITY

The probability the event B occurs, given that event A has happened, is represented as

$$P(B \mid A)$$

This is read as "the probability of B given A"

example



Suppose we have a bag containing 3 red balls and 2 green balls.
We want to find the probability of drawing two red balls.
Let's denote the events as follows:

Event A: Drawing a red ball on the first draw.
Event B: Drawing a red ball on the second draw.

Given that we've drawn a red ball on the first draw, there are now 4 balls left in the bag, 2 of which are red and 2 are green.

Now, since we have 2 red balls and 4 balls total left in the bag, the probability of drawing a red ball on the second draw, given that the first ball drawn is red, is $\frac{2}{4} = \frac{1}{2}$.

So the probability of B given A, denoted as $P(B \mid A)$ is $\frac{1}{2}$.

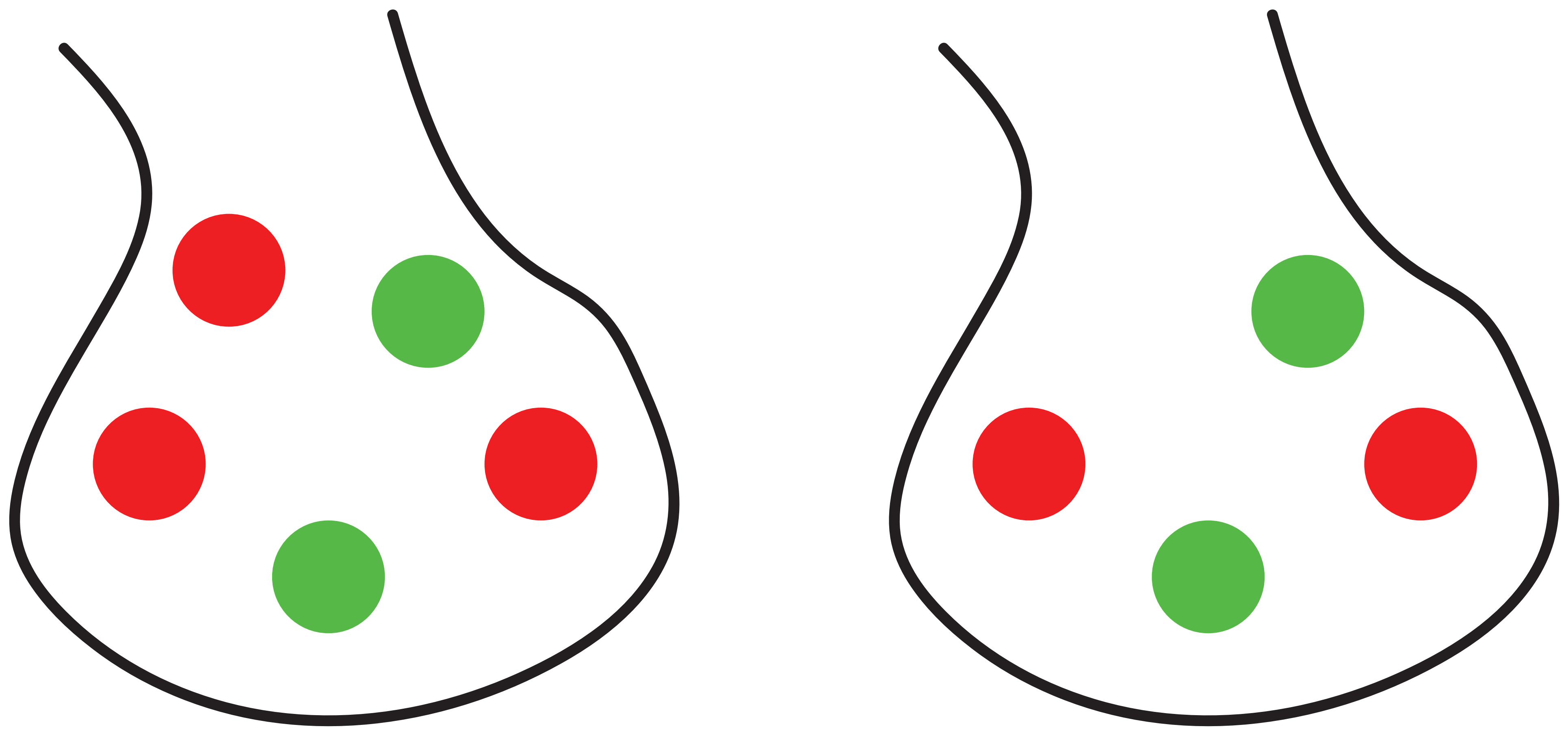
CONDITIONAL PROBABILITY FORMULA

If Events A and B are not independent, then

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

returning to previous example



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The probability of both events A and B occurring is:

$$P(A \cap B) = P(A) \cdot P(B \mid A) = \frac{2}{5} \cdot \frac{1}{2} = \frac{2}{10} = \frac{1}{5} .$$

example

You have just developed a new COVID-19 diagnostic test with your team in the lab.

Event A represents the event that the person actually has COVID-19.
Event B represents the event that the COVID-19 test comes back positive.

The prevalence of COVID-19 in a certain population might be $P(A)=0.02$, meaning that 2% of people in the population have COVID-19.

The sensitivity of the COVID-19 test might be $P(B|A)=0.95$, meaning that given a person has COVID-19, there is a 95% chance that the COVID-19 test will come back positive.

Q:What is the probability of that the person has COVID-19 and that the test comes back as positive?

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$$P(A \cap B) = P(A) \cdot P(B \mid A) = (0.02) \cdot (0.95) = 0.019$$

question

Suppose a health study examines the relationship between exercise frequency and the likelihood of developing certain health conditions. The data collected is summarized in the table below:

	No health condition	Has health condition	Total
Exercises regularly	800	200	1000
Does not exercise regularly	300	300	600
Total	1100	500	1600

Q. What is the probability that a randomly chosen individual has a health condition given that they exercise regularly?

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$$P(\text{has health condition} \mid \text{exercises regularly}) = \frac{200}{1000} = \frac{1}{5}$$

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BAYES' THEOREM

If Events A and B are not independent, then

$$P(A \mid B) = \frac{P(A)\,P(B \mid A)}{P(A)\,P(B \mid A) + P(\overline{A})\,P(B \mid \overline{A})}$$

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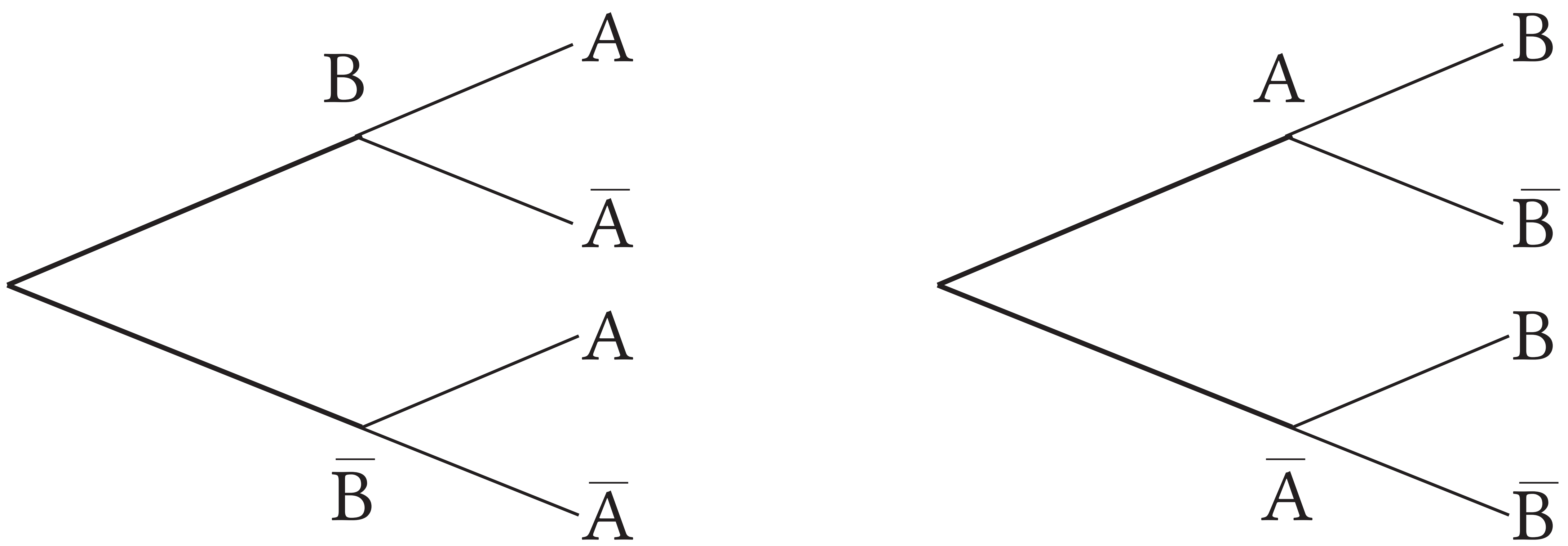

example

In a certain community, the prevalence of individuals being in close contact with an infected individual of a certain disease is 20%.
Among those who have been in close contact with an infected individual, the probability of contracting the disease is 30%. Among those who have **not** been in close contact with an infected individual, the probability of contracting the disease is 10%.
If a randomly selected individual from the community is found to have the disease,
what is the probability that they had close contact with an infected individual?

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Event B: Individual has the infectious disease.

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Event A: Individual had close contact with an infected individual.
Event B: Individual has the infectious disease.

$P(A) = 0.20$ $\quad$ $P(\overline{A}) = 0.80$
$P(B \mid A) = 0.30$
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$P(B \mid \overline{A}) = 0.10$

$$P(A \mid B) = \frac{0.20 \cdot 0.30}{0.20 \cdot 0.30 + 0.80 \cdot 0.10} = 0.4286$$

question (Monty Hall Problem)

Suppose you are a contestant on a game show. The game involves three doors.
Behind one of the doors is a car, and behind the other two doors are goats.
You pick a door, say Door 1, but before it's opened, the host, who knows what's behind each door, opens another door, say Door 3, revealing a goat. Now, the host offers you the opportunity to switch your choice to Door 2. Should you switch?

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A: The car is behind Door 1 (your initial choice).
B: The host opens Door 3 to reveal a goat.

The probability of the car being behind any specific door initially is $P(A) = \frac{1}{3}$.
Given that the car is behind Door 1, the probability that the host opens Door 3, revealing a goat is $P(B|A)=1$, because the host will always open a door with a goat behind it.
Given that the car is **not** behind Door 1, the probability that the host opens Door 3 (revealing a goat) is $P(B|\bar{A})=1$, because the host will always open a door with a goat behind it.

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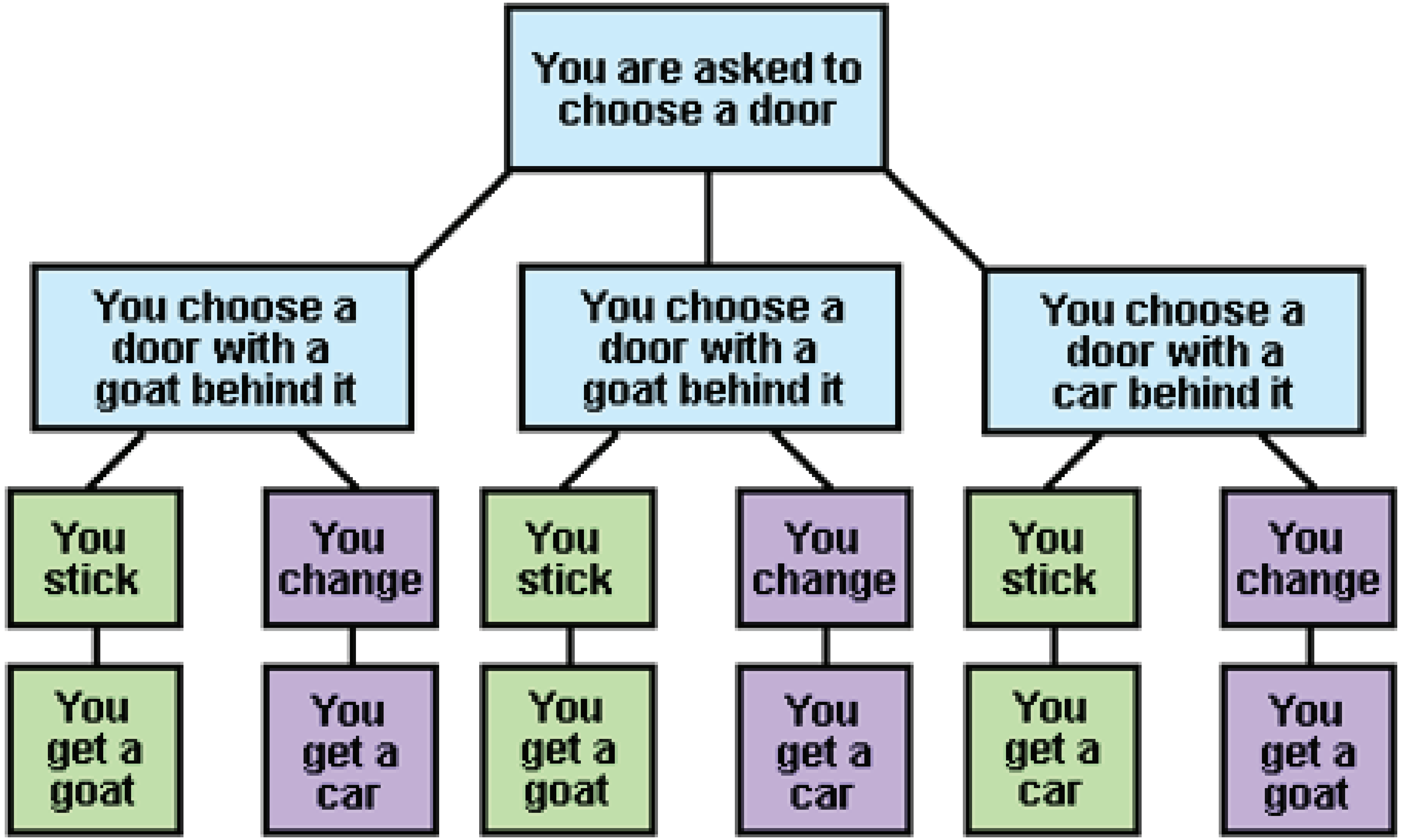
Given that the car is **not** behind Door 1, the probability that the host opens Door 3 (revealing a goat) is $P(B|\overline{A})=1$, because the host will always open a door with a goat behind it.

$$P(A \mid B) = \frac{P(A)\,P(B \mid A)}{P(A)\,P(B \mid A) + P(\overline{A})\,P(B \mid \overline{A})} = \frac{\frac{1}{3} \cdot 1}{\frac{1}{3} \cdot 1 \quad + \quad \frac{2}{3} \cdot 1} = \frac{1}{3}$$

So the probability that the car is behind Door 1 is unaffected by the host opening Door 3.
But the car has to be behind either Door 1 or Door 2, so P(winning the car if you change) = $\frac{2}{3}$

question (Monty Hall Problem)



BASIC COUNTING RULE

If we are asked to choose one item from each of two separate categories where there are m items in the first category and n items in the second category, then the total number of available choices is $\mathbf{m \cdot n}$.

This is sometimes called the multiplication rule for probabilities.

example

Suppose you have 5 different colored shirts (red, blue, green, yellow, and black) and 4 different colored pants (orange, purple, gray, and brown) in your wardrobe.

You want to select one shirt and one pair of pants to wear for the day.

Total number of different outfits:

#(shirts)·#(pants)

$= 5 \cdot 4 = 20$

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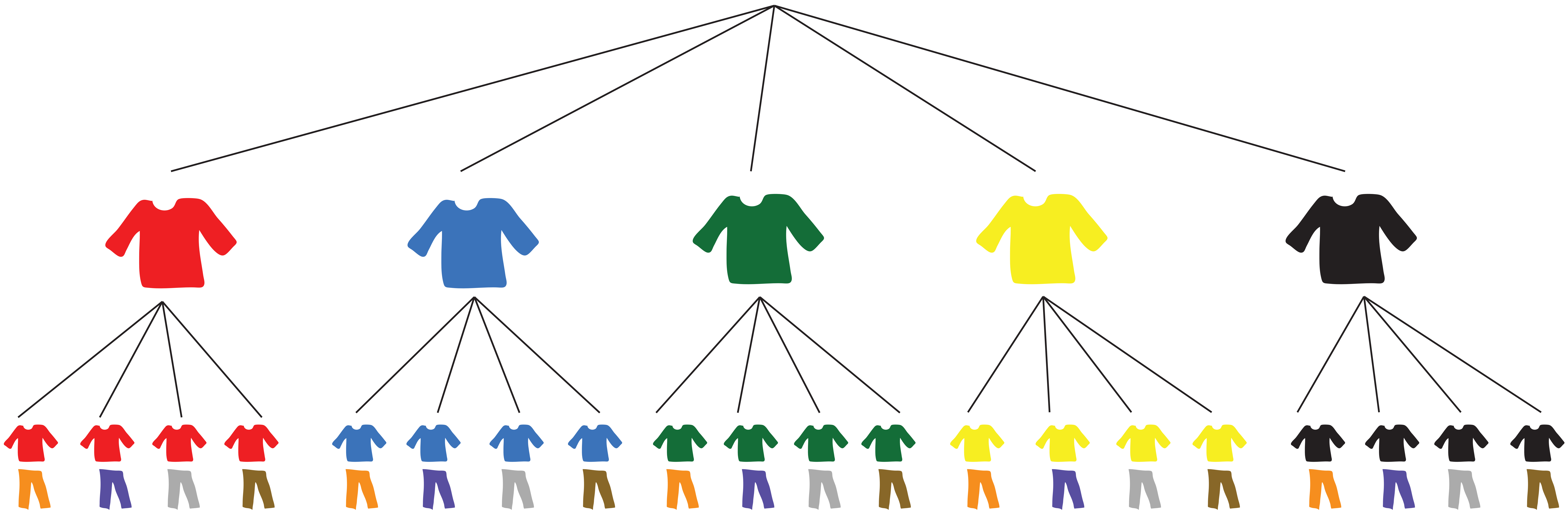

example

Let's consider a scenario where you're organizing a sports event with different activities and teams.

Suppose you have the following options:

3 types of activities (football, basketball, volleyball)
4 teams (Team A, Team B, Team C, Team D)
2 time slots (morning, afternoon)

Total number of different combinations: #(options for activities) × #(options for teams) × #(options for time slots)
$= 3 \cdot 4 \cdot 2 = 24$

Therefore, there are 24 different combinations considering the type of activity, team, and time slot.

question

You roll a dice 3 three times. how many possibilities are there in total?

question

You roll a dice 3 three times. how many possibilities are there in total?

Each roll has 6 possibilities.

total number of combinations $= 6 \cdot 6 \cdot 6 = 216$

question

how many different ways can we order the numbers

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad ?$$

question

how many different ways can we order the numbers

1 2 3 4 5 ?

1 2 3 4 5	2 1 3 4 5	3 1 2 4 5	4 1 2 3 5	5 1 2 3 4
1 2 3 5 4	2 1 3 5 4	3 1 2 5 4	4 1 2 5 3	5 1 2 4 3
1 2 4 3 5	2 1 4 3 5	3 1 4 2 5	4 1 3 2 5	5 1 3 2 4
1 2 4 5 3	2 1 4 5 3	3 1 4 5 2	4 1 3 5 2	5 1 3 4 2
1 2 5 3 4	2 1 5 3 4	3 1 5 2 4	4 1 5 2 3	5 1 4 2 3
1 2 5 4 3	2 1 5 4 3	3 1 5 4 2	4 1 5 3 2	5 1 4 3 2
1 3 2 4 5	2 3 1 4 5	3 2 1 4 5	4 2 1 3 5	5 2 1 3 4
1 3 2 5 4	2 3 1 5 4	3 2 1 5 4	4 2 1 5 3	5 2 1 4 3
1 3 4 2 5	2 3 4 1 5	3 2 4 1 5	4 2 3 1 5	5 2 3 1 4
1 3 4 5 2	2 3 4 5 1	3 2 4 5 1	4 2 3 5 1	5 2 3 4 1
1 3 5 2 4	2 3 5 1 4	3 2 5 1 4	4 2 5 1 3	5 2 4 1 3
1 3 5 4 2	2 3 5 4 1	3 2 5 4 1	4 2 5 3 1	5 2 4 3 1
1 4 2 3 5	2 4 1 3 5	3 4 1 2 5	4 3 1 2 5	5 3 1 2 4
1 4 2 5 3	2 4 1 5 3	3 4 1 5 2	4 3 1 5 2	5 3 1 4 2
1 4 3 2 5	2 4 3 1 5	3 4 2 1 5	4 3 2 1 5	5 3 2 1 4
1 4 3 5 2	2 4 3 5 1	3 4 2 5 1	4 3 2 5 1	5 3 2 4 1
1 4 5 2 3	2 4 5 1 3	3 4 5 1 2	4 3 5 1 2	5 3 4 1 2
1 4 5 3 2	2 4 5 3 1	3 4 5 2 1	4 3 5 2 1	5 3 4 2 1
1 5 2 3 4	2 5 1 3 4	3 5 1 2 4	4 5 1 2 3	5 4 1 2 3
1 5 2 4 3	2 5 1 4 3	3 5 1 4 2	4 5 1 3 2	5 4 1 3 2
1 5 3 2 4	2 5 3 1 4	3 5 2 1 4	4 5 2 1 3	5 4 2 1 3
1 5 3 4 2	2 5 3 4 1	3 5 2 4 1	4 5 2 3 1	5 4 2 3 1
1 5 4 2 3	2 5 4 1 3	3 5 4 1 2	4 5 3 1 2	5 4 3 1 2
1 5 4 3 2	2 5 4 3 1	3 5 4 2 1	4 5 3 2 1	5 4 3 2 1

FACTORIAL

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$$

question

Suppose there are 5 different tasks (A, B, C, D, E) to be assigned to 5 employees (Alice, Bob, Charlie, David, Emma) in a company.

How many ways can the tasks be assigned to the employees?

question

In a deck of 52 playing cards, how many different ways can you draw three cards in a specific order without replacement?

$$nPr = n \cdot (n-1) \cdot (n-2) \cdots (n-r+1)$$

We say that there are nPr permutations of size r that may be selected from among n choices without replacement when order matters.
It turns out that we can express this result more simply using factorials.

$$nPr = \frac{n!}{(n-r)!}$$

question

I have twelve different types of plants, and I want to arrange only five of them in a row on my garden bed. How many different ways could I do this?

COMBINATIONS

$$nCr = \frac{n!}{(n-r)!\ r!}$$

example

How many ways can I choose 3 socks from a drawer containing 23 socks?

question

In a group of 10 students, how many different ways can we choose a committee of 3 students to represent the class?

question

A pizza restaurant offers 10 different toppings. How many different combinations of 5 toppings can a customer choose for their pizza?