EVENTS AND OUTCOMES

The result of an experiment is called an **outcome**.

An **event** is any particular outcome or group of outcomes.

A **simple event** is an event that cannot be broken down further

The **sample space** is the set of all possible simple events.

Example

If we flip a fair coin twice, describe the sample space, a simple event and compound event.

The sample space is the set of all possible simple events: {HH, HT, TH, TT}

Example of a Simple event:
We flip two tails: {TT}

Example of a Compound event:
The first flip is a head: {HT, HH}

BASIC PROBABILITY

Given that all outcomes are equally likely, we can compute the probability of an event E using this formula:

$$P(E) = \frac{\text{Number of outcomes corresponding to the event E}}{\text{Total number of equally likely outcomes}}$$

Example

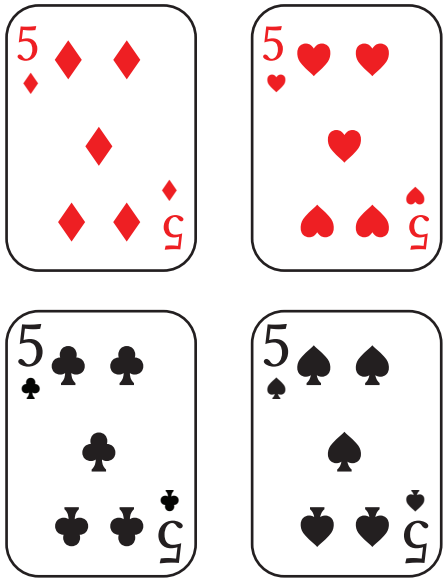
If we select a card from a standard deck of 52 cards, calculate:

P(picking a 5) =

Example

If we select a card from a standard deck of 52 cards, calculate:

$$P(\text{picking a 5}) = \frac{\text{number of 5's in the deck}}{\text{number of cards in the deck}}$$



$$= \frac{4}{52} = \frac{1}{13}$$

Example

If we randomly select a card from a standard deck of 52 playing cards, calculate:

P(♥) =

P(face) =

Example

If we randomly select a card from a standard deck of 52 playing cards, calculate:

$$P(\heartsuit) = \frac{\text{number of } \heartsuit \text{ in deck}}{\text{total number of cards in deck}}$$

$$= \frac{13}{52} = \frac{1}{4}$$

$$P(\text{face}) = \frac{\text{number of faces in deck}}{\text{total number of cards in deck}}$$

$$= \frac{12}{52} = \frac{3}{13}$$

Question

If we randomly draw a marble from a bag containing 5 red marbles, 3 blue marbles, and 2 green marbles, calculate:

P(drawing a red marble)

P(drawing a green or blue marble)

Question

If we randomly draw a marble from a bag containing 5 red marbles, 3 blue marbles, and 2 green marbles, calculate:

$$P(\text{drawing a red marble}) = \frac{\text{number of red marbles}}{\text{total number of marbles}} =$$

$$P(\text{drawing a green or blue marble}) = \frac{\text{number of green and blue marbles}}{\text{total number of marbles}} =$$

Question

At some random moment, you glance at a calendar in the month of October.

a. What is the probability that the day is the 10th?

b. What is the probability that the day is the 10th or after?

Question

Compute the probability of randomly drawing one card from a deck and getting a Queen.

CERTAIN AND IMPOSSIBLE EVENTS

An **impossible** event has a probability of 0.

A **certain event** has a probability of 1.

The probability of any event must be:

$$0 \leq P(E) \leq 1$$

CERTAIN AND IMPOSSIBLE EVENTS

The **complement of an event** is the event "E doesn't happen".

The notation $\bar{E}$ is used for the complement of event E.

We can compute the probability of the complement using $P(\bar{E}) = 1 - P(E)$
Notice also that $P(E)=1 - P(\bar{E})$

Question

What is the probability that a card drawn from a deck is not a Jack?

Question

A box contains 12 balls: 4 red, 5 blue, and 3 green. A ball is drawn randomly from the box. Find the probability of the following events:

The ball drawn is blue.
The probability is:

The ball drawn is not blue.
The probability is:

Question

What is the probability that Alice goes on vacation not in summer?
(Assume equal probability of each month and only one month is chosen)

INDEPENDENT EVENTS

Events A and B are **independent** events if the probability of Event B occurring is the same whether or not Event A occurs.

Examples of independent events

Flipping a fair coin twice

Rolling a fair six-sided die and flipping a fair coin

Selecting a marble from a bag and then selecting another marble from the same bag with replacement

Question

Are the following events independent or dependent?

Randomly selecting two cards from a standard deck without replacement.

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Are the following events independent or dependent?

Randomly selecting two cards from a standard deck without replacement.

The probability of the second draw is dependent on the outcome of the first draw is a card has been removed and cannot be chosen again.

Question

Are the following events independent or dependent?

Life expectancy and where you live in New York City.

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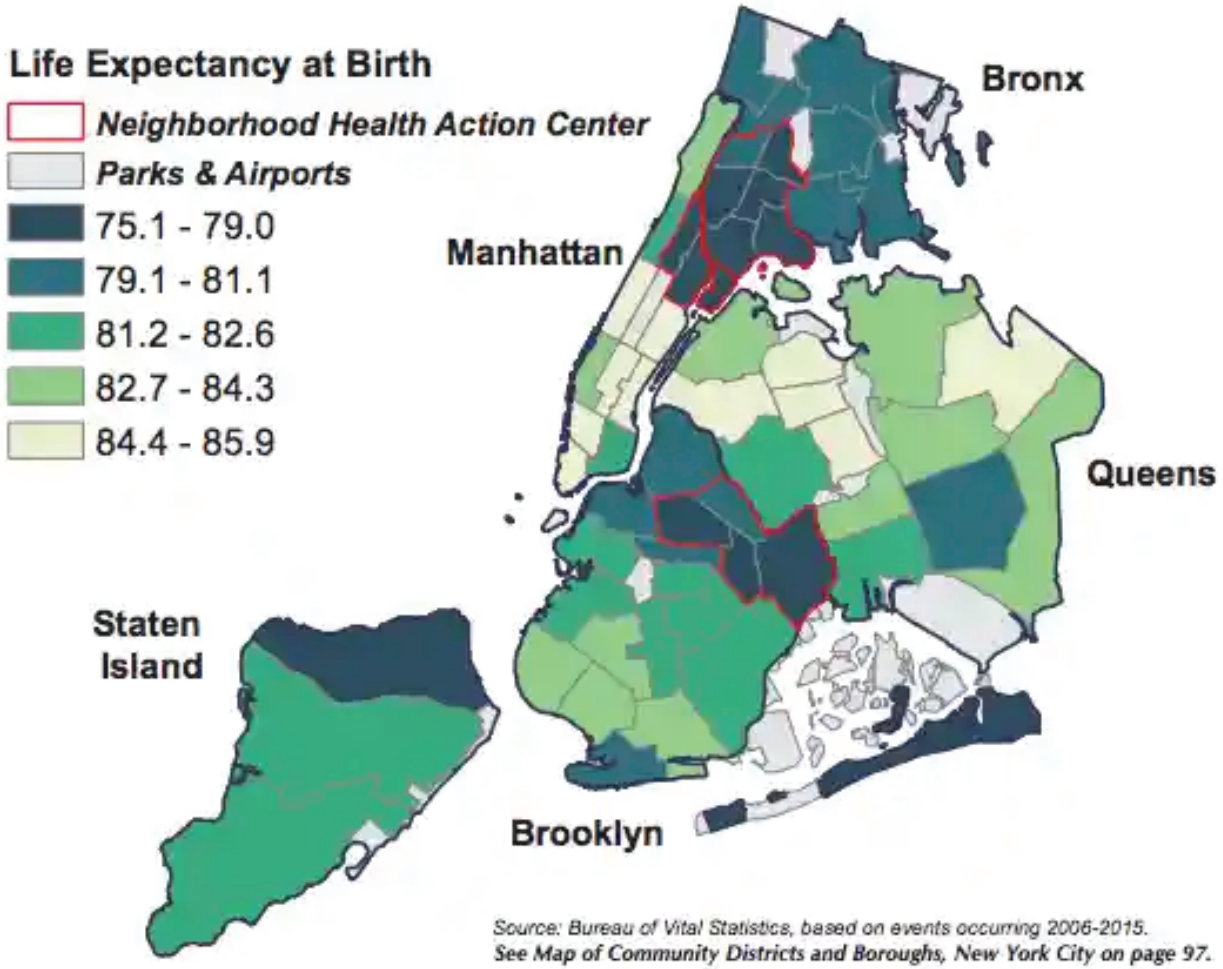
Life expectancy and where you live in New York City.

The cohort life expectancy is the average life length of a particular cohort – a group of individuals born in a given year.

LIFE EXPECTANCY

Figure 4. Life Expectancy at Birth by Community District, New York City, 2006-2015



Source: Bureau of Vital Statistics, based on events occurring 2006-2015.
See Map of Community Districts and Boroughs, New York City on page 97.

- In 2015, New York City's life expectancy at birth was highest in Murray Hill (85.9), the Upper East Side (85.9), Battery Park/ Tribeca (85.8), Greenwich Village/SOHO (85.8), and Elmhurst/Corona (85.6).
- In 2015, life expectancy at birth was lowest in Brownsville (75.1), Morrisania (76.2), Central Harlem (76.2), The Rockaways (76.5), and Bedford Stuyvesant (76.8).

P(A AND B) FOR INDEPENDENT EVENTS

If events A and B are independent, then the probability of both A and B occurring is:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

where P(A and B) is the probability of events A and B both occurring, P(A) is the probability of event A occurring, and P(B) is the probability of event B occurring.

Question

What is the probability of rolling a five followed by a six when rolling a die?

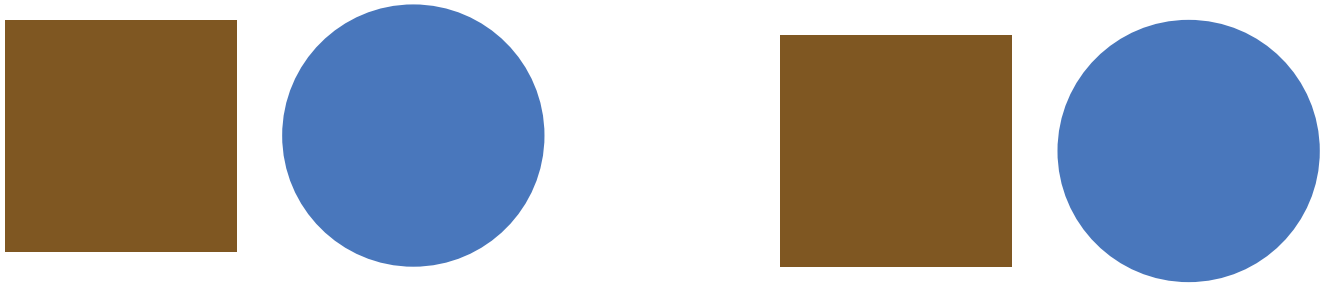
Question

What is the probability of rolling a five followed by a six when rolling a die?

$$\begin{aligned} P(\text{five and six}) &= P(\text{five}) \cdot P(\text{six}) \\ &= \frac{1}{6} \cdot \frac{1}{6} \\ &= \frac{1}{36} \end{aligned}$$

Question

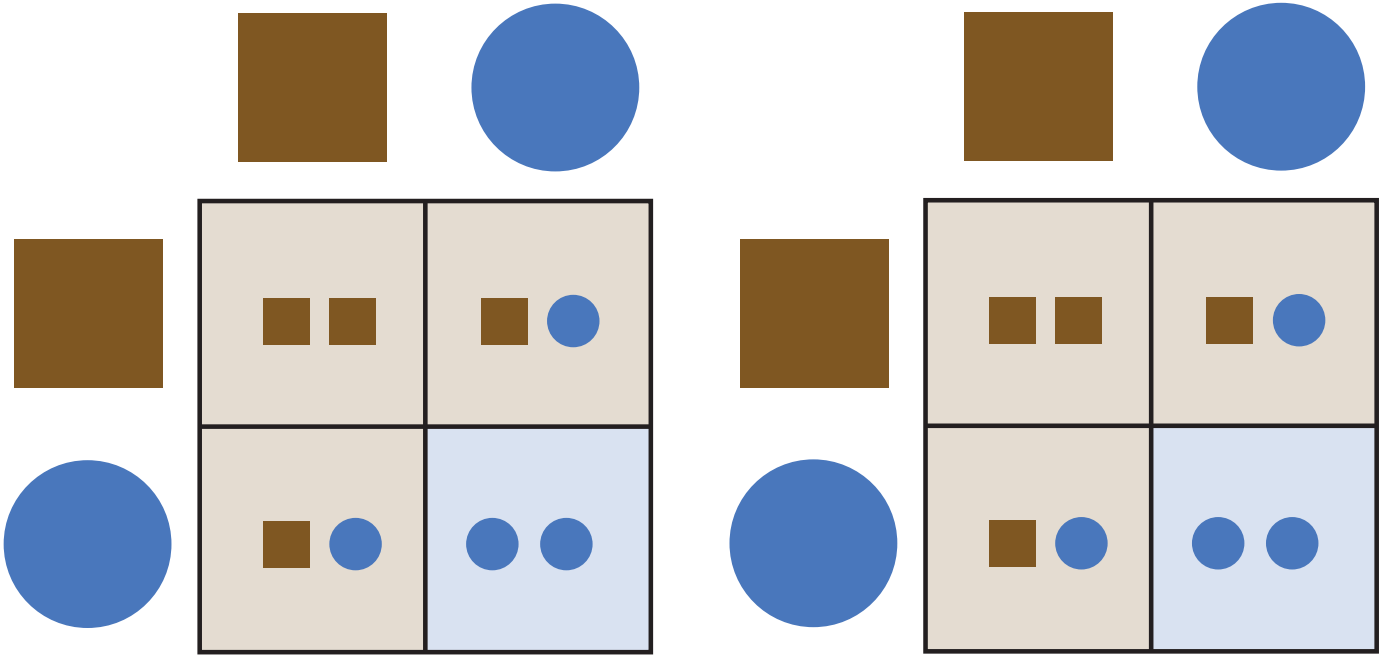
What is the probabilty of the parents



having one child with brown eyes and another child with
blues eyes?

Question

What is the probabilty of the parents  having one child with brown eyes and another child with blues eyes?



$$P(\text{brown and blue}) = P(\text{brown}) \cdot P(\text{blue})$$

$$= \frac{3}{4} \cdot \frac{1}{4}$$

$$= \frac{3}{16}$$

P(A OR B)

The probability of either A or B occurring (or both) is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example

In a group of 100 students, 60 students play tennis (event A) and 45 students play basketball (event B). Among them, 30 students play both tennis and basketball. What is the probability that a randomly selected student plays either tennis or basketball?

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In a group of 100 students, 60 students play tennis (event A) and 45 students play basketball (event B). Among them, 30 students play both tennis and basketball. What is the probability that a randomly selected student plays either tennis or basketball?

$$P(\text{tennis or basketball}) = P(\text{tennis}) + P(\text{basketball}) - P(\text{tennis and basketball})$$

$$= \frac{60}{100} + \frac{45}{100} - \frac{30}{100} = \frac{75}{100}$$

Question

What is the probability that we draw either an odd numbered card in a deck of cards or a ten?

Question

What is the probability that we draw either an odd numbered card in a deck of cards or a ten?

$$P(\text{odd or 10}) = P(\text{odd}) + P(\text{ten}) - P(\text{odd and ten})$$

$$= \frac{20}{52} + \frac{4}{52} - \frac{0}{52} = \frac{24}{52} = \frac{6}{13}$$

MUTUALLY EXCLUSIVE

Two events A and B are mutually exclusive if

$$P(A \text{ or } B) = P(A) + P(B)$$